

SUPPLEMENT TO FOREST SCIENCE, VOL. 32, NO. 2

JUNE 1986

(ISSN 0015-749X)

A PUBLICATION OF THE SOCIETY OF AMERICAN FORESTERS

***A Growth and Yield Model  
for Thinned Stands of  
Yellow-Poplar***

BY

BRUCE R. KNOEBEL

HAROLD E. BURKHART

DONALD E. BECK

***Forest Science***

***Monograph 27***

***1986***

**PUBLISHED BY SOCIETY OF AMERICAN FORESTERS**

**WASHINGTON, D. C.**

FOREST SCIENCE MONOGRAPHS are published by the Society of American Foresters as supplements to *Forest Science*. It is intended that these publications will accommodate the longer and more comprehensive articles devoted to forestry research.

Papers of 32 printed pages or longer (approximately 20,000 words) will be considered for publication as FOREST SCIENCE MONOGRAPHS. The same editor and advisory board will govern acceptance of papers as for *Forest Science*.

Correspondence concerning manuscripts and other editorial matters should be addressed to Dr. Harold E. Burkhardt, School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061. Correspondence concerning remittances, orders for additional copies, and inquiries concerning the status of manuscripts accepted for publication should be addressed to the Society of American Foresters, 5400 Grosvenor Lane, Bethesda, MD 20814.

The monographs are published intermittently as separates, but distributed following regular issues of *Forest Science* free to subscribers (unless charges are required by the sponsor). The monographs are listed in the table of contents of the appropriate issue of *Forest Science*, and included in the annual index.

*Editor*

**Harold E. Burkhardt**

*School of Forestry  
and Wildlife Resources  
Virginia Polytechnic Institute  
and State University  
Blacksburg, VA 24061*

*Staff Editor*

**William P. Everard**

*Society of American Foresters*

(ISSN 0015-749X)

*Printed in the United States of America  
1986*

A PUBLICATION OF THE SOCIETY OF AMERICAN FORESTERS

***A Growth and Yield Model  
for Thinned Stands of  
Yellow-Poplar***

BY

BRUCE R. KNOEBEL

HAROLD E. BURKHART

DONALD E. BECK

*Forest Science*

*1986*

*Monograph 27*

# CONTENTS

Introduction .....	1
Literature review .....	2
Stand-level models .....	2
Diameter distribution models .....	2
Model development .....	4
Plot data .....	4
Stand-level component .....	4
Stand table generation .....	9
Parameter recovery procedure .....	9
Stand attribute prediction .....	11
Thinning algorithm .....	14
Tree volume equations .....	15
Computer program .....	16
Input data .....	16
Stand attribute prediction .....	16
Estimation of Weibull parameters .....	17
Stand table derivation .....	17
Thinning the stand table .....	17
Example regime .....	18
Model evaluation .....	18
Evaluation of whole stand estimates .....	18
Predicted stand tables .....	20
Size class distributions .....	20
Volume yields .....	20
Effect of thinning regime on yield .....	21
Weight of thinning .....	21
Number of thinnings .....	21
Timing of thinning .....	28
Discussion .....	29
Model limitations and recommendations .....	29
Summary .....	36
Literature Cited .....	37
Appendices .....	40
Appendix 1. Example run of yellow-poplar growth and yield program ..	40
Appendix 2. Flow chart diagram of yellow-poplar growth and yield pro-	
gram .....	47
Appendix 3. Source code for yellow-poplar growth and yield program ..	48

# *A Growth and Yield Model for Thinned Stands of Yellow-Poplar*

BRUCE R. KNOEBEL

HAROLD E. BURKHART

DONALD E. BECK

**ABSTRACT.** Simultaneous growth and yield equations were developed for predicting basal area growth and cubic-foot volume growth and yield in thinned stands of yellow-poplar. A joint loss function involving both volume and basal area was used to estimate the coefficients in the system of equations. The estimates obtained were analytically compatible, invariant for projection length, and numerically equivalent with alternative applications of the equations. Given estimates of basal area and cubic-foot volume from these equations, board-foot volumes can also be calculated.

As an adjunct to the stand-level equations, compatible stand tables were derived by solving for the parameters of the Weibull distribution from attributes predicted with the stand-level equations. This procedure for estimating the parameters of the diameter distributions of the stands before thinning gave reasonable estimates of number of trees, basal area, and cubic-foot volume per acre by diameter class. The thinning algorithm removes a proportion of the basal area from each diameter class and produces stand and stock tables after thinning from below that are consistent with those generated before thinning.

**ADDITIONAL KEY WORDS.** *Liriodendron tulipifera*, mensuration, thinning, modeling.

---

## INTRODUCTION

IN THE EASTERN UNITED STATES, yellow-poplar (*Liriodendron tulipifera* L.) is an important commercial species that is cut primarily for lumber and veneer. Because tree size and quality greatly influence yields of these products, thinning is an important silvicultural tool in yellow-poplar management. Most stands of yellow-poplar can produce a number of lumber- and veneer-size trees without thinning; however, thinning concentrates growth on the best and largest trees. Reliable estimates of stand growth and yield are needed to determine optimal thinning regimes.

Beck and Della-Bianca (1972) published equations for predicting basal area growth and cubic-foot volume growth and yield in yellow-poplar stands thinned to various levels of basal area. However, flexible models that supply information about the diameter distributions—and hence product distributions—are needed to better evaluate the effects and results of various thinning options.

The objectives of this study were to develop a growth and yield model for yellow-poplar that can be used to evaluate thinning options. This model should be efficient to use and provide detailed information about stand structure. To accomplish these objectives, we

1. Developed a stand-level model for thinned stands of yellow-poplar, and
2. Derived diameter distributions from predicted stand attributes.

---

The authors are, respectively, former Graduate Research Assistant (now employed by Eastman Kodak Company, Rochester, New York); Thomas M. Brooks Professor, Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061; and Project Leader, USDA Forest Service, Southeastern Forest Experiment Station, Asheville, North Carolina 28804. Manuscript received 22 February 1984.

## LITERATURE REVIEW

### *Stand-Level Models*

The first yield predictions in the United States were made using normal yield tables for natural even-aged stands of a given species. Temporary plots in stands of "normal" stocking were used to construct these tables through graphical techniques. Volume and yield tables of this type for yellow-poplar in the southern Appalachians were presented by McCarthy (1933).

MacKinney and others (1937) suggested the use of multiple regression to construct variable-density yield equations. Subsequently, MacKinney and Chaiken (1939) used a multiple regression analysis to construct a yield prediction equation for loblolly pine stands. Since that time, many investigators have used multiple regression to construct stand aggregate growth and/or yield expressions (Schumacher and Coile 1960; Coile and Schumacher 1964; Goebel and Warner 1969; Burkhart and others 1972a, 1972b; and others).

Until the early 1960's, independent equations were developed to predict growth and yield, often resulting in inconsistent and illogical results. Buckman (1962) introduced a model for red pine where yield was obtained through mathematical integration of the growth equation over time, thus taking into account the logical relationship which should exist between growth and yield equations. Clutter (1963) discussed this concept of compatibility between growth and yield prediction in detail and developed a compatible growth and yield model for natural loblolly pine stands.

Sullivan and Clutter (1972) refined Clutter's equations to develop a simultaneous growth and yield model for loblolly pine that provided not only analytically, but also numerically consistent growth and yield predictions. This growth and yield model has been successfully used for loblolly pine (Brender and Clutter 1970, Sullivan and Williston 1977, Murphy and Sternitzke 1979, Burkhart and Sprinz 1984), shortleaf pine (Murphy and Beltz 1981), slash pine (Bennett 1970), and yellow-poplar (Beck and Della-Bianca 1972).

### *Diameter Distribution Models*

Stand yields have also been predicted using diameter distribution analysis procedures. In such cases it is often assumed that the underlying diameter distribution of the stand can be adequately characterized by a probability density function (pdf).

Clutter and Bennett (1965) fitted the beta distribution to observed diameter frequency data from old-field slash pine plantations, and, from this, developed variable density stand tables. Bennett and Clutter (1968) used these stand tables to estimate multiple-product yields for slash pine plantations. The parameters of the beta distribution that approximated the diameter distribution were predicted from stand variables (age, site index, and density). The number of trees and volume per acre in each diameter class were then calculated, and per acre yield estimates were obtained by summing over the diameter classes of interest.

Following these same procedures, McGee and Della-Bianca (1967) successfully fitted the beta distribution to describe diameter distributions in even-aged natural stands of yellow-poplar. From this diameter distribution information, Beck and Della-Bianca (1970) then obtained yield estimates for even-aged stands of unthinned yellow-poplar. A similar approach was used for loblolly pine plantations by Lenhart and Clutter (1971), Lenhart (1972), and Burkhart and Strub (1974). In each of these cases, the minimum and maximum diameters defining the limits of the distributions, as well as the pdf parameters, were predicted from functions of stand characteristics.

The beta distribution is very flexible in shape and can approximate a wide range of diameter distributions. In addition, the pdf has finite limits which constrain all diameters to be within upper and lower bounds. A disadvantage of this distribution, however, is that the pdf must be numerically integrated to obtain probabilities over various ranges of the random variable, i.e., to obtain the proportion of trees in each diameter class, as the cumulative distribution function (cdf) does not exist in closed form.

More recently, the Weibull distribution has been widely applied for describing diameter distributions. The pdf is flexible in shape, the parameters are reasonably easy to estimate, and the cdf exists in closed form—a major advantage over the beta pdf. The Weibull pdf exists in either a two or three parameter form, the three parameter pdf having the advantage of increased flexibility.

First used as a diameter distribution model by Bailey (1972), the Weibull distribution has been applied to a wide range of situations. For example, it has been used to describe diameter distributions in loblolly pine plantations (Smalley and Bailey 1974a, Schreuder and Swank 1974, Feduccia and others 1979, Cao and others 1982, Amateis and others 1984), slash pine plantations (Dell and others 1979, Bailey and others 1982), shortleaf pine plantations (Smalley and Bailey 1974b), longleaf pine plantations (Lohrey and Bailey 1976), natural stands of loblolly pine (Burk and Burkhart 1984), and white pine (Schreuder and Swank 1974). Bailey and Dell (1973) concluded no other distribution proposed exhibited as many desirable features as the Weibull.

Given an appropriate density function, Strub and Burkhart (1975) presented a class-interval-free method for obtaining yield estimates over specified diameter class limits. The general equation form is given by

$$V = N \int_l^u g(D)f(D) dD$$

where

- $V$  = expected stand volume per unit area,
- $N$  = number of trees per unit area,
- $D$  = dbh,
- $g(D)$  = individual tree volume equation,
- $f(D)$  = pdf for  $D$ , and
- $l, u$  = lower and upper merchantability limits, respectively, for the product described by  $g(D)$ .

Using attributes from a whole stand model and the relationship given by the class-interval-free equation presented by Strub and Burkhart (1975), Hyink (1980) introduced a method of solving for the parameters of a pdf approximating the diameter distribution. The approach was to predict stand average attributes of interest for a specified set of stand conditions, and use these estimates as a basis to "recover" the parameters of the underlying diameter distribution using the method of moments technique.

When constructed independently, even from the same data set, stand average and diameter distribution models, which give different levels of resolution, do not necessarily produce the same estimates of stand yield for a given set of stand conditions (Daniels and others 1979). The advantages of the procedure outlined by Hyink are ability to partition total yield by diameter class, mathematical compatibility between the whole stand and diameter distribution based yield models, and consistency among the various stand yield estimates.

Based on this procedure, Frazier (1981) developed a method to approximate

the diameter distributions of unthinned plantations of loblolly pine from whole stand predictions of stand attributes using the beta and Weibull pdf's. Using the same concept, Matney and Sullivan (1982) developed a model for thinned and unthinned loblolly pine plantations. Cao and others (1982) used the Weibull function to derive diameter distributions from predicted stand attributes for thinned loblolly pine plantations. Cao and Burkhart (1984) used a similar approach with a segmented Weibull cumulative distribution to derive empirical diameter distributions from predicted stand attributes for thinned loblolly pine plantations. Hyink and Moser (1983) extended the idea and developed a generalized framework for projecting forest yield and stand structure using diameter distributions.

## MODEL DEVELOPMENT

Several desirable properties were sought when deriving a growth and yield model for thinned stands of yellow-poplar. In particular, we wanted the equations to exhibit analytic compatibility between growth and yield, invariance for projection length, and numeric equivalency between alternative applications of the equations. In addition to whole stand volume and basal area, we also wanted to derive stand tables to provide flexibility for evaluating the full range of utilization options. Consequently, another goal was to derive stand tables that are compatible with the whole stand values.

The model for thinned stands of yellow-poplar was developed in two stages. In the first stage, equations to predict stand-level attributes were obtained. In the second stage, stand tables were derived from the whole-stand attributes by solving for parameters in a theoretical diameter distribution model (in this case the Weibull distribution was used) while ensuring compatibility between the whole stand and diameter distribution estimates of the stand-level attributes.

### *Plot Data*

Data for this study were collected by the U.S. Forest Service, Southeastern Forest Experiment Station, from 141 circular,  $\frac{1}{4}$ -acre plots established in the Appalachian Mountains of North Carolina (93 plots), Virginia (31 plots), and Georgia (17 plots). The plots contained 75 percent or more yellow-poplar in the overstory, were free from insect and disease damage, and showed no evidence of past cutting (Beck and Della-Bianca 1972).

Each plot was thinned (using low thinning) at the time of installation to obtain a range of basal areas for different site-age combinations. Site index at age 50 was determined for each plot with an equation published by Beck (1962). Volumes and basal areas were computed when the plots were thinned and again after five growing seasons. At the time of initial plot establishment, the stands ranged from 17 to 76 years in age, 74 to 138 feet in site index (base age 50 years), and 44 to 209 sq ft per acre in basal area.

Table 1 shows a summary of the plot data before and after the first thinning (measure 1), before and after the second thinning (measure 2), 5 years after the second thinning (measure 3), and 10 years after the second thinning (measure 4). Basal area and cubic-foot volume growth between the four measurement periods are presented in Table 2.

### *Stand-Level Component*

When fitting the stand-level components, we used the models of Beck and Della-Bianca (1972) as a starting point because these models exhibit desirable properties and they were successfully fitted to the first 5-year growth data from the yellow-poplar plots. Beck and Della-Bianca fitted the following models (adapted from



Sullivan and Clutter 1972) for prediction of basal area and cubic volume at some projected age when site index, initial age, and basal area are given:

$$\ln(Y_2) = b_0 + b_1(S^{-1}) + b_2(A_2^{-1}) + b_3(A_1/A_2)\ln(B_1) + b_4(1 - A_1/A_2) + b_5(S)(1 - A_1/A_2) \quad (1)$$

where

$Y_2$  = stand volume per unit area at some projected age,  $A_2$   
 $S$  = site index,  
 $B_1$  = present basal area per unit area, and  
 $A_1$  = present age.

When  $A_2 = A_1 = A$  and  $B_2 = B_1 = B$ , equation (1) reduces to the general yield model

$$\ln(Y) = b_0 + b_1(S^{-1}) + b_2(A^{-1}) + b_3\ln(B). \quad (2)$$

The yield prediction model (1) was derived by substituting a basal area projection equation for the basal area term in the general yield model (2). Therefore, inserting  $\ln(Y_2)$ ,  $A_2$ , and  $\ln(B_2)$  into equation (2) and setting the resulting expression equal to the right side of equation (1) and solving the equality for  $\ln(B_2)$  gives the basal area projection model

$$\ln(B_2) = (A_1/A_2)\ln(B_1) + (b_4/b_3)(1 - A_1/A_2) + (b_5/b_3)(S)(1 - A_1/A_2). \quad (3)$$

Beck and Della-Bianca (1972) used ordinary least squares to estimate the coefficients in (1) and substituted the ratios  $b_4/b_3$  and  $b_5/b_3$  as parameter estimates in the basal area projection equation (3) to ensure that exact numerical equivalency would result when projecting future volume from (1) and when projecting future basal area from (3) and solving for future volume by substitution of appropriate values into (2).

In our analyses, equation (1) was fitted by ordinary least squares to each of the growth periods and standard  $F$ -tests were performed to determine if separate coefficients were needed for each period or if data from some of the periods could be combined. From these tests, we determined that two sets of coefficients were needed—one for the growth period after one thinning and a second for the growth periods following two thinnings. The second thinning apparently altered stand structure and vigor so that growth relationships were significantly affected.

After determining that separate coefficients were needed for the growth periods following one thinning and following two thinnings, final estimates of the parameters in the volume and basal area projection equations were computed by using a simultaneous fitting procedure. This procedure, applied previously by Burkhart and Sprinz (1984) to data from thinned loblolly pine plantations, involves minimizing the loss function:

$$F = \frac{\sum_i (Y_i - \hat{Y}_i)^2}{\hat{\sigma}_Y^2} + \frac{\sum_i (B_i - \hat{B}_i)^2}{\hat{\sigma}_B^2} \quad (4)$$

where

$Y_i$  and  $\hat{Y}_i$  = observed and predicted volume values, respectively,  
 $B_i$  and  $\hat{B}_i$  = observed and predicted basal area values, respectively,  
 $\hat{\sigma}_Y^2$  and  $\hat{\sigma}_B^2$  = estimates of the variance about the regression lines for volume and basal area, respectively, computed as the mean square error from ordinary least squares fits of equations (1) and (3).

TABLE 1. Yellow-poplar plot data summary.

Time of measure <sup>a</sup> and stand variable <sup>b</sup>	Number of plots	Minimum value	Mean value	Maximum value
Measure 1				
Age	141	17	46.9	76
Site		74	107.8	138
Ntb		104	231.8	432
Nta		32	105.1	340
Ntr		12	126.7	312
Bab		44	134.8	209
Baa		25	85.4	153
Bar		2	49.5	137
Cvb		1,336	5,772.2	11,171
Cva		1,106	3,857.8	8,102
Cvr		48	1,881.0	6,275
Bvb		493	18,671.9	55,078
Bva		329	14,418.2	41,140
Bvr		0	4,253.6	27,624
Measure 2				
Age	141	22	51.9	81
Site		74	107.8	138
Ntb		32	105.1	340
Nta		28	83.5	256
Ntr		0	21.6	108
Bab		38	97.4	171
Baa		22	86.0	150
Bar		0	11.4	36
Cvb		1,224	4,588.7	9,398
Cva		722	4,112.6	8,109
Cvr		0	476.1	1,438
Bvb		199	18,221.3	48,852
Bva		198	16,963.7	41,813
Bvr		0	1,257.5	7,039
Measure 3				
Age	140	27	57.1	86
Site		74	107.7	138
Ntb		28	81.6	256
Nta		28	81.6	256
Ntr		0	0	0
Bab		31	97.6	164
Baa		31	97.6	164
Bar		0	0	0
Cvb		1,222	4,889.9	9,030
Cva		1,222	4,889.9	9,030
Cvr		0	0	0
Bvb		2,018	21,455.9	46,742
Bva		2,018	21,455.9	46,742
Bvr		0	0	0
Measure 4				
Age	138	33	62.4	91
Site		74	107.6	138
Ntb		28	80.7	248
Nta		28	80.7	248
Ntr		0	0	0

TABLE 1. Continued.

Time of measure <sup>a</sup> and stand variable <sup>b</sup>	Number of plots	Minimum value	Mean value	Maximum value
Bab		40	110.0	178
Baa		40	110.0	178
Bar		0	0	0
Cvb		1,565	5,621.3	10,070
Cva		1,565	5,621.3	10,070
Cvr		0	0	0
Bvb		3,482	25,771.3	51,275
Bva		3,482	25,771.3	51,275
Bvr		0	0	0

<sup>a</sup> Plot data before and after first thinning (measure 1), before and after second thinning (measure 2), 5 years after second thinning (measure 3), and 10 years after second thinning (measure 4).

<sup>b</sup> Age = age of stand (years).

Site = site index (feet, base age 50 years).

Ntb = number of trees/ac prior to thinning.

Nta = number of trees/ac after thinning.

Ntr = number of trees/ac removed in thinning.

Bab = basal area (sq ft/ac) prior to thinning.

Baa = basal area (sq ft/ac) after thinning.

Bar = basal area (sq ft/ac) removed in thinning.

Cvb = cubic-foot volume/ac prior to thinning.

Cva = cubic-foot volume/ac after thinning.

Cvr = cubic-foot volume/ac removed in thinning.

Bvb = board-foot volume/ac prior to thinning.

Bva = board-foot volume/ac after thinning.

Bvr = board-foot volume/ac removed in thinning.

Beginning with coefficients estimates from the ordinary least squares fit of (1), the coefficients of models (1) and (3) were adjusted through an iterative process until  $F$  in the loss function was minimized. This process of simultaneously fitting the two models (with the imposed restriction that the coefficients in the basal area equation are equal to the appropriate ratios of the volume equation coefficients) results in a system of equations that are compatible and numerically consistent. Different weights could be assigned to the two components, but we felt that for management decisions involving thinning equal weight should be given to both volume and basal area projection. The simultaneous estimation procedure is more statistically efficient (in that the basal area growth information is used in the fitting) and produces more stable estimates of the basal area equation coefficients for varying units of measure and merchantability standards in (1) than does the derivation of coefficients in (3) from the least squares fit of (1) (Burkhart and Sprinz 1984). The basal area and cubic-foot volume equations from the simultaneous fitting procedure and their related fit statistics are presented in Tables 3 and 4. In the evaluation process, current volume yield values (i.e., observations for which  $A_2 = A_1 = A$ ) were used in addition to the growth data, thus doubling the number of yield observations. Due to the model structure, current basal area values could not be used.

Beck and Della-Bianca (1975) predicted the ratio of board-foot volume to basal area using dominant stand height and residual quadratic mean stand diameter. In this study, we developed the following equation from the plot data to relate board-foot volume to stand basal area and cubic-foot volume.

TABLE 2. Summary of basal area and cubic-foot volume growth during the 5-year periods between the four plot measurements.

Growth period	Variable <sup>a</sup>	Minimum value	Mean value	Maximum value	Mean annual growth
5 years after first thinning	B1	25	85.4	153	2.4
	B2	38	97.4	171	
	Bg	5	12.0	33	
	V1	1,106	3,857.8	8,102	158.9
	V2	1,224	4,588.7	9,398	
	Vg	318	794.7	1,920	
5 years after second thinning	B1	22	86.0	150	2.5
	B2	31	97.6	164	
	Bg	4	12.5	32	
	V1	722	4,112.6	8,109	158.1
	V2	1,222	4,889.9	9,030	
	Vg	260	790.7	2,190	
10 years after second thinning	B1	31	97.6	164	2.6
	B2	40	110.0	178	
	Bg	-1	12.9	26	
	V1	1,222	4,889.9	9,030	171.4
	V2	1,565	5,621.3	10,070	
	Vg	-61	856.8	1,740	

<sup>a</sup> B1 = basal area (sq ft/ac) at beginning of growth period.

B2 = basal area (sq ft/ac) at end of growth period.

Bg = B2 - B1, i.e., 5 years growth.

V1 = cubic-foot volume/ac at beginning of growth period.

V2 = cubic-foot volume/ac at end of growth period.

Vg = V2 - V1, i.e., 5 years growth.

TABLE 3. Simultaneous growth and yield equations<sup>a</sup> for prediction of total cubic-foot volume and basal area per acre.

$$\ln(Y_2) = b_0 + b_1(S^{-1}) + b_2(A_2^{-1}) + b_3(A_1/A_2)\ln(B_1) + b_4(1 - A_1/A_2) + b_5(S)(1 - A_1/A_2)$$

$$\ln(B_2) = (A_1/A_2)\ln(B_1) + (b_4/b_3)(1 - A_1/A_2) + (b_5/b_3)(S)(1 - A_1/A_2)$$

For stands thinned once	For stands thinned twice
$b_0 = 5.35740$	$b_0 = 5.33115$
$b_1 = -102.45728$	$b_1 = -97.95286$
$b_2 = -21.95901$	$b_2 = -25.19324$
$b_3 = 0.97473$	$b_3 = 0.98858$
$b_4 = 4.11893$	$b_4 = 5.84476$
$b_5 = 0.01293$	$b_5 = 0.00018$

<sup>a</sup> Where

$Y_2$  = predicted total cubic-foot volume per acre at projected age,  $A_2$ .

$A_1$  = initial age.

$S$  = site index, base age 50 years (feet).

$B_1$  = initial basal area per acre (sq ft).

$B_2$  = predicted basal area per acre (sq ft) at  $A_2$ .

$\ln$  = natural (Napierian) logarithm.

TABLE 4. Fit statistics for evaluating cubic-foot volume and basal area prediction from the simultaneous growth and yield equations.

Equation	Number of observations	Minimum residual value <sup>a</sup>	Mean residual value	Mean absolute residual value	Maximum residual value	Standard deviation of residual values	R <sup>2b</sup>
Cubic-foot volume	840	-808.91	6.68	156.46	1,250.39	219.74	0.9865
Basal area	419	-13.66	.78	2.90	16.62	3.69	.9860

<sup>a</sup> A residual value is the difference between the observed and predicted value of the dependent variable:  $r_i = Y_i - \hat{Y}_i$ .

<sup>b</sup> The R<sup>2</sup> value was computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^n r_i^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

where

$Y_i$  =  $i^{\text{th}}$  observed value of the dependent variable.

$\hat{Y}_i$  =  $i^{\text{th}}$  predicted value of the dependent variable.

$\bar{Y}$  = mean value of the dependent variable.

$r_i$  =  $i^{\text{th}}$  residual value as defined above in footnote a.

$n$  = number of observations.

$$\begin{aligned} \text{BFV} &= 1363.09165 - 306.96647(B) + 10.26187(\text{CFV}) \\ R^2 &= 0.9730 \quad s = 1785.1 \end{aligned} \quad (5)$$

where

BFV = board-foot volume per acre, International 1/4-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob) (1-foot stump).

B = basal area per acre (sq ft) of all stems.

CFV = total cubic-foot volume per acre.

R<sup>2</sup> = coefficient of determination.

s = root mean square error.

Given equations for estimating the total stand cubic volume and basal area, the board-foot volume of a selected portion of the stand according to an 8-inch top diameter outside bark can be estimated. This approach does not allow sufficient flexibility, however, to account for rapidly changing utilization standards. Thus an extremely valuable adjunct to the overall stand values is a stand table. When computing a stand table it is important that it be logically and consistently related to the overall stand characteristics.

### Stand Table Generation

#### PARAMETER RECOVERY PROCEDURE

The parameter recovery procedure introduced by Hyink (1980) and further discussed and developed by Frazier (1981), Matney and Sullivan (1982), Cao and others (1982), Hyink and Moser (1983), and Cao and Burkhart (1984) was used to obtain estimates of the parameters of the Weibull pdf, which was used to describe the diameter distributions of yellow-poplar stands before and after thinning. The recovery method was selected because it provides compatible whole stand and diameter distribution estimates of specified stand attributes.

The Weibull pdf exists in either a two or three parameter form. These two forms are defined as follows. Three parameter Weibull density

$$f_Z(z; a, b, c) = \begin{cases} \left(\frac{c}{b}\right) \left(\frac{z-a}{b}\right)^{c-1} \exp\left[-\left(\frac{z-a}{b}\right)^c\right] & a, b, c > 0 \\ 0, \text{ otherwise.} & z > a \end{cases}$$

Two parameter Weibull density

$$f_X(x; b, c) = \begin{cases} \left(\frac{c}{b}\right) \left(\frac{x}{b}\right)^{c-1} \exp\left[-\left(\frac{x}{b}\right)^c\right] & y, b, c > 0 \\ 0, \text{ otherwise} \end{cases}$$

where

- $a$  = the location parameter,
- $b$  = the scale parameter,
- $c$  = the shape parameter,
- $Z$  = the random variable (diameter), and
- $X = Z - a$ .

With the general diameter distribution yield function,

$$Y_i = N \int_l^u g_i(x) f(x; \theta) dx \quad (6)$$

where

- $Y_i$  = total per unit area value of the stand attribute defined by  $g_i(x)$
- $g_i(x)$  = stand attribute as a function of  $x$
- $f(x; \theta)$  = pdf for  $x$
- $N$  = number of trees per unit area
- $l, u$  = lower and upper diameter limits, respectively, for the product described by  $g_i(x)$ ,

integration over the range of diameters,  $X$ , for any  $g_i(x)$ , gives the total per unit area value of the stand attribute defined by  $g_i(x)$ . Average diameter, basal area per acre, and total cubic volume per acre are examples of such stand attributes. The number of stand attribute equations must equal the number of parameters to be estimated in order to solve the system of equations for recovery of the pdf parameters.

Letting  $g_i(x)$  equal  $x^i$ , one obtains the  $i^{\text{th}}$  noncentral moment of  $X$  as

$$E(X_i) = \int_{-\infty}^{\infty} X^i f(x; \theta) dx$$

and the parameter recovery system is simply the method of moments technique of pdf parameter estimation (Mendenhall and Scheaffer 1973).

In the case of forest diameter distributions, the first noncentral moment,  $E(X)$ , is estimated by

TABLE 5. Equations for prediction of the first and second noncentral moments of the diameter distribution.<sup>a</sup>

$\ln(B_2) = (A_1/A_2)\ln(B_1) + (b_4/b_3)(1 - A_1/A_2) + (b_5/b_3)(S)(1 - A_1/A_2)$ (from Table 4)			
$\ln(\bar{d}^2 - \bar{d}^2) = b_0 + b_1\ln(B) + b_2\ln(H_d) + b_3(A \cdot N)/1,000$			
<i>For before first thinning</i>		<i>For after first thinning</i>	
$b_0 = -13.40824$	$R^2 = 0.8133$	$b_0 = -5.20164$	$R^2 = 0.3726$
$b_1 = 0.45213$	$s^2 = 0.09357$	$b_1 = 0.80773$	$s^2 = 0.2225$
$b_2 = 3.05978$		$b_2 = 0.72383$	
$b_3 = -0.20664$		$b_3 = -0.33560$	
$\bar{d} = \{B/(0.005454N) - \exp[\ln(\bar{d}^2 - \bar{d}^2)]\}^{1/2}$			
$\ln(D_{\min}) = 1.19439 + 0.05637[B/(0.005454N)]^{1/2} + 3.04022/(N^{1/2}) - 394.07219/(A \cdot H_d)$			
$R^2 = 0.8251 \quad s^2 = 0.02045$			
(For all measures except before first thinning where $D_{\min}$ is set equal to 5.0 inches.)			

<sup>a</sup> Where

- $A_1$  = stand age at beginning of projection period.
- $A_2$  = stand age at end of projection period.
- $A$  = stand age.
- $B_1$  = basal area/acre (sq ft) at beginning of projection period.
- $B_2$  = basal area/acre (sq ft) at end of projection period.
- $B$  = basal area/acre (sq ft)
- $S$  = site index, base age 50 years.
- $\bar{d}^2$  = average squared tree dbh of stand (inches<sup>2</sup>).
- $\bar{d}$  = average tree dbh of stand (inches).
- $H_d$  = average height of dominant and codominant trees of stand (feet).
- $N$  = number of trees/acre.
- $D_{\min}$  = minimum dbh of stand (inches).
- $R^2$  = coefficient of determination.
- $s^2$  = mean squared error.
- $\ln$  = natural (Napierian) logarithm.

$$\sum x_i/N = \bar{x},$$

the arithmetic mean diameter of the stand, and the second noncentral moment,  $E(X^2)$ , is the estimated by

$$\sum x_i^2/N = \bar{x}^2 = \text{basal area/acre}/0.005454N,$$

(the quadratic mean diameter of the stand) where  $N$  is the number of trees per acre. Hence, the first two moments of the diameter distribution have stand-level interpretations that are common in forestry practice.

Stand average estimates of the first  $k$  moments produce a system of  $k$  equations with  $k$  unknown parameters which can be solved to obtain estimates of the pdf parameters while ensuring compatibility between whole stand and diameter distribution estimates of the stand attributes described by the moment equations.

#### STAND ATTRIBUTE PREDICTION

Regression equations used to obtain estimates of the first two noncentral moments, and subsequently solve for the parameters of the Weibull distribution, are given in Table 5.

The moment-based system of equations for the three parameter Weibull distribution led to convergence problems and the three parameter Weibull pdf was reduced to the two parameter form using the transformation  $X = Z - a$ . That is,

the location parameter  $a$  was set equal to a constant or predicted outside the system of equations, depending on stand characteristics.

Because independent estimates of average diameter,  $\bar{d}$ , and average squared diameter,  $\overline{d^2}$ , often produced illogical crossovers and hence negative variances (i.e.,  $\overline{d^2} - \bar{d}^2 < 0$ ), a procedure discussed by Frazier (1981) was used, i.e., the logarithm of the variance of the diameters,  $\ln(\overline{d^2} - \bar{d}^2)$ , was predicted. Given a value of  $\overline{d^2}$  obtained from the estimate of basal area and the estimate of  $\ln(\overline{d^2} - \bar{d}^2)$ ,  $\bar{d}$  was determined algebraically.

As only those trees  $\geq 4.5$  inches in dbh were tallied, and due to the extremely small variability in minimum stand diameters for the plot data prior to the first thinning, the minimum diameter,  $D_{min}$ , was set equal to 5.0 inches in stands prior to the first thinning.

Bailey and Dell (1973) state that  $a$  can be considered the smallest possible diameter in the stand. An approximation to this smallest possible diameter is given by  $D_{min}$ , the minimum observed diameter on the sample plots. This value is positively biased since  $D_{min}$  is always greater than or equal to the true smallest diameter in the stand. Thus the value of  $a$  should most likely be  $0 \leq a \leq D_{min}$ . Five values for  $D_{min}$  were selected and sensitivity analyses conducted. Using values of 0,  $\frac{1}{3}(D_{min})$ ,  $\frac{1}{2}(D_{min})$ ,  $\frac{2}{3}(D_{min})$ , and  $D_{min}$  for  $a$ , and the recovered estimates of  $b$  and  $c$ , observed and predicted diameter distributions were compared. As was previously found by Frazier (1981) for thinned loblolly pine stands, preliminary tests with the yellow-poplar data indicated that the  $a$  parameter of the Weibull distribution could be estimated reasonably well from the minimum stand diameter,  $D_{min}$ , as

$$a = 0.5(D_{min}).$$

The two equations for the two parameter system are

$$\bar{x} = \int_0^\infty xf(x; b, c) dx = b\Gamma(1 + 1/c) \quad (7)$$

$$\overline{x^2} = \int_0^\infty x^2f(x; b, c) dx = b^2\Gamma(1 + 2/c). \quad (8)$$

The estimated variance of the distribution is given by

$$s^2 = \overline{x^2} - \bar{x}^2 = b^2[\Gamma(1 + 2/c) - \Gamma^2(1 + 1/c)] \quad (9)$$

and the coefficient of variation (CV) is estimated by

$$CV = \frac{s}{\bar{x}} = \frac{[\Gamma(1 + 2/c) - \Gamma^2(1 + 1/c)]^{1/2}}{\Gamma(1 + 1/c)}. \quad (10)$$

Given estimates of  $\bar{x}$  and  $\overline{x^2}$ , the coefficient of variation is a function of  $c$  alone, thus reducing the order of the system. Under this formulation, there exists a unique solution for  $c$ , and simple iterative techniques for solving one equation in one unknown can be used to obtain a value for  $c$ . With  $c$  known,  $b$  is solved from  $\bar{x} = b\Gamma(1 + 1/c)$ , and  $a$  is estimated with a constant or equation external to the system. In a sense, this is a "hybrid" system in that it combines the parameter-prediction and parameter-recovery systems.

When applying the system, the same stand-level basal area equation is used when deriving diameter distributions and when estimating overall stand basal area in order to ensure compatibility between the two levels of stand detail.

The computer program written by Frazier (1981) to approximate the diameter



TABLE 6. Stand attribute prediction equations.<sup>a</sup>

$\ln(H_d/H) = -0.09675 + (1/D - 1/D_{\max}) \cdot [3.70051 - 0.02828 \ln(B) - 138.35633(A^{-1}) + 0.04010(S)]$ $R^2 = 0.8312 \quad s^2 = 0.006037$ $\text{TVOB} = 0.010309 + 0.002399(D^2 \cdot H)$ $\ln(B) = b_0 + b_1(A^{-1}) + b_2(S) + b_3(N^{-1})$			
<i>For before first thinning</i>		<i>For after first thinning</i>	
$b_0 = 4.55808$	$R^2 = 0.6838$	$b_0 = 4.16240$	$R^2 = 0.7404$
$b_1 = -31.21173$	$s^2 = 0.02493$	$b_1 = -38.13602$	$s^2 = 0.03980$
$b_2 = 0.01324$		$b_2 = 0.01606$	
$b_3 = -77.35908$		$b_3 = -47.19922$	
$\ln(N) = b_0 + b_1(A^{-1}) + b_2(S) + b_3(B^{-1})$		<i>For after second thinning</i>	
$b_0 = 6.43346$	$R^2 = 0.6115$	$b_0 = 4.24861$	$R^2 = 0.7929$
$b_1 = 38.24834$	$s^2 = 0.03671$	$b_1 = -45.83883$	$s^2 = 0.02634$
$b_2 = -0.01309$		$b_2 = 0.01566$	
$b_3 = -67.25874$		$b_3 = -37.78880$	
<i>For before first thinning</i>		<i>For after first thinning</i>	
$b_0 = 6.43346$	$R^2 = 0.6115$	$b_0 = 6.12444$	$R^2 = 0.7707$
$b_1 = 38.24834$	$s^2 = 0.03671$	$b_1 = 59.93859$	$s^2 = 0.06980$
$b_2 = -0.01309$		$b_2 = -0.01911$	
$b_3 = -67.25874$		$b_3 = -73.59987$	
<i>For before second thinning</i>		<i>For after second thinning</i>	
$b_0 = 6.43346$	$R^2 = 0.6115$	$b_0 = 6.12335$	$R^2 = 0.7213$
$b_1 = 38.24834$	$s^2 = 0.03671$	$b_1 = 69.03772$	$s^2 = 0.07113$
$b_2 = -0.01309$		$b_2 = -0.02083$	
$b_3 = -67.25874$		$b_3 = -78.12201$	

<sup>a</sup> Where

- $H_d$  = average height of dominant and codominant trees of stand (feet).
- $H$  = total tree height (feet).
- $D$  = dbh (inches).
- $D_{\max}$  = maximum dbh of stand (inches).
- $B$  = basal area/acre (sq ft) of stand.
- $A$  = age of stand.
- $S$  = site index, base age 50 years (feet).
- $\text{TVOB}$  = total tree cubic-foot volume, outside bark.
- $N$  = number of trees/acre of stand.
- $R^2$  = coefficient of determination.
- $s^2$  = mean squared error.
- $\ln$  = natural (Napierian) logarithm.

distributions of unthinned plantations of loblolly pine was used as a framework in the development of the yellow-poplar growth and yield program. Equations to predict stand attributes required by the solution routine, such as mean height of the dominant and codominant trees, number of trees per acre, and individual tree volume, are presented in Table 6.

The total height equation is a slight modification of the one presented by Beck and Della-Bianca (1970) with number of trees per acre replaced by basal area per acre. The tree volume equation is of the same form presented by Beck (1963) and was fitted using weighted least squares procedures.

#### THINNING ALGORITHM

Using the equations presented in Table 6, diameter distributions before and after the first thinning were predicted for 10 randomly selected sample plots to observe the "goodness-of-fit" of the system and also to check for logical consistencies which should exist between stand tables for thinned and unthinned conditions.

Although the predicted distributions closely approximated the observed distributions, some discrepancies were present among the stand tables of the thinned and unthinned plots. Predicted numbers of trees increased in some diameter classes after thinning, and, in some instances, the thinned stand table had a larger maximum stand diameter and/or a smaller minimum stand diameter than those in the corresponding unthinned stand table. It was apparent that the diameter distribution predictions before and after a thinning from below could not be carried out independently, but had to be conditioned such that the previously stated inconsistencies could not occur.

As an alternative to two independent predictions, the diameter distribution prior to thinning was predicted, as before, then a proportion of the basal area in each diameter class was removed to simulate the thinning. With this procedure it is impossible for the number of trees in a given class to increase as trees can only be removed from a class. Consequently, minimum diameter can only increase and maximum diameter can only decrease, if they change at all.

A function was defined specifying the amount of basal area to be removed from each diameter class. The following equation form relating the proportion of basal area removed in a diameter class to the ratio of the midpoint diameter of the class to the average squared diameter of the stand was used to "thin" the predicted stand table.

$$P_i = \exp[b_1(d_i^2/\bar{d}^2)^{b_2}] \quad (11)$$

where

- $P_i$  = proportion of basal area removed from diameter class  $i$ ,
- $d_i$  = midpoint diameter of class  $i$ ,
- $\bar{d}^2$  = average squared diameter of stand, and
- $b_1, b_2$  = coefficients estimated from the data.

As the plot data were taken from stands thinned from below, the removal function "thins" more heavily in the smaller diameter classes than in the larger diameter classes. Equation (11), when fitted, represents the average removal pattern in the data used to estimate the parameters. Separate removal equations were fitted for stands after the first and second thinnings due to the obvious differences in the size-class distributions. Coefficient estimates and fit statistics for the two equations are given in Table 7.

Once the basal area removal functions were defined, the thinning algorithm was as follows:

TABLE 7. Coefficient estimates and fit statistics for the basal area removal function.<sup>a</sup>

$P_i = \exp[b_1(d_i^2/\bar{d}^2)^{b_2}]$	
For first thinning	For second thinning
$b_1 = -0.70407$	$b_1 = -2.61226$
$b_2 = 1.87666$	$b_2 = 2.00627$
$R^2 = 0.5614$	$R^2 = 0.4060$
MSE = 0.0843	MSE = 0.0672

<sup>a</sup> Where

$P_i$  = proportion of basal area removed from diameter class  $i$ .

$d_i$  = midpoint diameter of class  $i$ .

$\bar{d}^2$  = average squared diameter of class  $i$ .

MSE = mean square error.

$$R^2 = 1 - \frac{\sum_{i=1}^n (P_i - \hat{P}_i)^2}{\sum_{i=1}^n (P_i - \bar{P})^2}$$

$\hat{P}_i$  = predicted value of  $P_i$ .

$\bar{P}$  = mean of the  $P_i$  values.

$n$  = sample size.

1. Predict the diameter distribution prior to thinning from the Weibull distribution.
2. Starting with the smallest diameter class, remove the proportion of basal area specified by the removal function.
3. Proceed through the diameter classes until the desired level of basal area to be removed is attained.
4. If the required basal area removal is not obtained after the largest diameter class is reached, return to the smallest diameter class and remove the remaining basal area in that class. Proceed in this manner through the diameter classes until the desired level of basal area removal is attained.

This procedure validated fairly well against the observed data where the thinnings from below produced stands that were thinned heavily in the lower diameter classes, and diameter distributions that were frequently left-truncated.

### Tree Volume Equations

As yellow-poplar is cut for a variety of products, reliable estimates of volume to any specified merchantable top diameter and/or height limit are essential. Beck (1963) published cubic-foot volume tables for yellow-poplar in the southern Appalachians based on diameter at breast height (dbh) and total tree height. Total height, rather than merchantable height, was used to estimate volume inside and outside bark to 4- and 8-inch top diameter limits. However, merchantability standards change rapidly and it is desirable to have a set of volume estimating equations that are completely general and flexible for obtaining estimates for any specified portion of tree boles. To provide estimates of cubic-foot volume to any desired top diameter or height limit while ensuring that the predicted volumes were logically related, we predicted total stem volume and the ratio of merchantable stem volume to total stem volume for any specified top diameter or height limit according to the methods described by Burkhart (1977) and Cao and Burk-

hart (1980). Information on the individual tree data analyses, which include taper functions as well as the volume equations, can be found in Knoebel and others (1984).

### *Computer Program*

The source code for the yellow-poplar growth and yield model, written in FORTRAN Level-G, is given in Appendix 3. The computer program is summarized and illustrated in a simplified flow chart diagram presented in Appendix 2. The steps and procedures outlined in the flow chart are discussed in the following sections.

#### INPUT DATA

The input data required by the program are:

- Age at beginning of projection period.
- Age at end of projection period (equal to age at beginning of projection period if no projection desired).
- Site index in feet (base age 50 ft).
- Basal area per acre at beginning of projection period (sq ft).
- Number of trees per acre at beginning of projection period.
- Number of previous thinnings.

Either basal area or number of trees per acre or both must be known. Given one measure of stand density, the other can be predicted from age, site index, and the known measure of stand density from equations fitted to the plot data. For projecting stands, the known number of trees or the number of trees obtained from a previously generated stand table should be entered. When this information is not known, the number of trees must be estimated.

#### STAND ATTRIBUTE PREDICTION

Given the input data, the following stand attributes are computed.

- Average height of the dominant and codominant trees in feet.
- Minimum diameter in inches.
- Arithmetic mean diameter in inches.
- Quadratic mean diameter in inches.

If stand-level estimates are desired, they are computed at this point.

- Number of trees per acre.
- Basal area per acre (sq ft).
- Total cubic-foot volume per acre.
- Board-foot volume per acre, International 1/4-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top (ob).

Once the stand-level attributes are generated and displayed, the user has the option to:

- Produce the corresponding stand/stock table,
- Make another projection, or
- Terminate the growth and yield program.

To obtain the corresponding stand/stock table, estimates of the Weibull distribution parameters must first be computed.

## ESTIMATION OF WEIBULL PARAMETERS

Given the input data and the predicted stand attributes, a computer solution routine developed by Burk and Burkhart (1984) is used to obtain estimates of the Weibull parameters. The routine solves a moment-based three parameter Weibull system of equations where the  $a$  parameter is predicted independent of the system.

## STAND TABLE DERIVATION

Given the parameter estimates, number of trees by diameter class are obtained by multiplying the total number of trees per acre by the proportion of the total number of trees in a given class as determined by the three parameter Weibull cdf. Basal area and cubic-foot volume by diameter class are obtained by numerically integrating the general diameter distribution yield function (6) with  $g_i(x)$  equal to  $0.005454(\text{dbh}^2)$  for basal area and  $g_v(x)$  equal to a total cubic-foot volume equation, which is a function of dbh alone, for cubic-foot volume. The numerical integration is carried out using a solution routine developed by Hafley and others (1982). Board-foot volumes in those diameter classes  $\geq 11$  inches are obtained according to the procedures described by Beck (1964). First, merchantable cubic-foot volume to an 8-inch top diameter (ob) is computed using the volume equations developed by Knoebel and others (1984). Then, using an equation presented by Beck, a board-foot/cubic-foot ratio, and, subsequently, a board-foot volume is calculated for a tree of a specified dbh. Given the number of trees by diameter class and this calculated board-foot volume per tree, an International  $\frac{1}{4}$ -inch board-foot volume for trees  $\geq 11$  inches dbh to an 8-inch top (ob) is computed by diameter class.

The user can substitute any total cubic-foot volume equation desired into the program provided all inputs for the equation are a function of diameter alone. For example, if total height is required in the volume equation, which is the case in this program, then an equation to predict total height as a function of dbh must also be supplied.

In addition to number of trees, basal area, and cubic-foot and board-foot volumes per acre by diameter class, the following stand attributes are also given.

- Input data
- Minimum diameter in inches
- Quadratic mean diameter in inches
- Maximum diameter in inches
- Average height of dominants and codominants in feet
- Total number of trees per acre
- Total basal area per acre in square feet
- Total cubic-foot volume per acre
- Total board-foot volume per acre, International  $\frac{1}{4}$ -inch rule for all trees in the 11-inch dbh class and above to an 8-inch top (ob).

## THINNING THE STAND TABLE

After the projected stand table and associated summary statistics are printed, the user has the option to thin the stand, in which case a residual basal area must be specified. Basal area is then removed from each diameter class according to the thinning algorithm described previously, until the residual basal area limit is met. The number of trees and the cubic-foot and board-foot volumes removed from a diameter class are obtained from the following equations.

$$Nr_i = Br_i / (0.005454 D_i^2)$$

$$CVr_i = (Nr_i / Np_i) CVp_i$$

$$BVR_i = (Nr_i / Np_i) BVP_i$$

where

- $Nr_i$  = number of trees removed from diameter class  $i$
- $Np_i$  = number of trees prior to thinning in diameter class  $i$
- $Br_i$  = basal area removed from diameter class  $i$
- $D_i$  = midpoint dbh of diameter class  $i$
- $CVr_i$  = cubic-foot volume removed from diameter class  $i$
- $CVp_i$  = cubic-foot volume prior to thinning in diameter class  $i$
- $BVR_i$  = board-foot volume removed from diameter class  $i$
- $BVP_i$  = board-foot volume prior to thinning in diameter class  $i$ .

As with the unthinned stand table, a similar stand attribute summary is given for the thinned stand table.

At this point, the user has the option to "rethin" the original predicted stand table to a different residual basal area. This can be done any number of times, to any level of residual basal area greater than zero and less than or equal to the original stand basal area. As before, once the stand/stock table is displayed, and the stand summary statistics are given, the user may either make another projection or terminate the growth and yield program.

#### EXAMPLE REGIME

An example run from the growth and yield model is given in Appendix 1 to illustrate the various options available and the output produced at each step of the program. The following thinning regime was used in the example.

##### Initial

conditions: Site index (base age 50) = 100 feet  
 Initial age = 20 years  
 Initial basal area = 80 sq ft/acre.

Regime: Thin to 50 sq ft/acre at age 20

Project to age 40 and thin to 70 and 80 sq ft/acre.

#### MODEL EVALUATION

##### *Evaluation of Whole Stand Estimates*

For each of the 141 sample plots, total basal area and cubic-foot volume per acre were computed by summing across the diameter classes of the generated stand tables. In each case, observed minus predicted basal area and cubic-foot volume per acre were calculated. Summary statistics, as well as an  $R^2$  value, were calculated for the basal area and cubic-foot volume residuals. These values are presented in Tables 8 and 9.

Bias, represented by the mean residual, decreases, and goodness-of-fit, represented by  $R^2$ , increases for both basal area and cubic-foot volume for the measurement periods after the first thinning, as opposed to the measurement prior to thinning. This may be due to the fact that the diameter distributions of the stands became smoother and more unimodal after the first thinning. Before the first thinning, diameter distributions were generally irregular and often multimodal,

TABLE 8. Summary statistics for the residual values representing observed minus predicted basal area per acre for the sample plot data.

Measurement period	Number of observations	Minimum residual value <sup>a</sup>	Mean residual value	Mean absolute residual value	Maximum residual value	Standard deviation of residual values	R <sup>2b</sup>
Before first thinning	141	0.07	3.64	3.64	26.45	3.13	0.9902
After first thinning	141	.02	.67	.67	2.26	.44	.9998
Before second thinning	141	.03	.73	.73	2.33	.45	.9998
After second thinning	141	.03	.69	.69	2.19	.47	.9998

<sup>a</sup> Residual value computed as the observed minus the predicted value of the dependent variable.  
 $r_i = Y_i - \hat{Y}_i$

<sup>b</sup> The R<sup>2</sup> value was computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^n r_i^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

where

$Y_i$  =  $i^{\text{th}}$  observed value of the dependent variable.  
 $\hat{Y}_i$  =  $i^{\text{th}}$  predicted value of the dependent variable.  
 $\bar{Y}$  = mean value of the dependent variable.  
 $r_i$  =  $i^{\text{th}}$  residual value, as defined above in footnote a.  
 $n$  = number of observations.

TABLE 9. Summary statistics for the residual values representing observed minus predicted total cubic-foot volume per acre for the sample plot data.

Measurement period	Number of observations	Minimum residual value <sup>a</sup>	Mean residual value	Mean absolute residual value	Maximum residual value	Standard deviation of residual values	R <sup>2b</sup>
Before first thinning	141	-399.13	206.94	249.21	970.32	232.86	0.9860
After first thinning	141	-783.53	-80.57	123.09	223.36	164.21	.9898
Before second thinning	141	-498.23	167.72	194.45	685.67	173.57	.9904
After second thinning	141	-498.23	151.55	173.94	685.67	151.34	.9920

<sup>a</sup> Residual value computed as the observed minus the predicted value of the dependent variable.  
 $r_i = Y_i - \hat{Y}_i$

<sup>b</sup> The R<sup>2</sup> value was computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^n r_i^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

where

$Y_i$  =  $i^{\text{th}}$  observed value of the dependent variable.  
 $\hat{Y}_i$  =  $i^{\text{th}}$  predicted value of the dependent variable.  
 $\bar{Y}$  = mean value of the dependent variable.  
 $r_i$  =  $i^{\text{th}}$  residual value, as defined above in footnote a.  
 $n$  = number of observations.

making modeling with a Weibull distribution difficult. As the thinnings "smoothed out" the distributions, the bias and goodness-of-fit generally improved. The smoothing effects of the thinnings are most noticeable with basal area as the parameter recovery solution procedure was conditioned on the basal area, and not on cubic-foot volume.

An evaluation of the parameter recovery procedure at the diameter class level was also conducted. Using the plot data and the predicted number of trees obtained from the solution routines, the observed and predicted number of trees by diameter class were computed for each plot.

A Chi-square goodness-of-fit statistic was calculated for each plot before and after the first thinning as well as before and after the second thinning. Calculated Chi-square statistics from the 141 plots exhibited trends similar to those found earlier at the whole stand level in that goodness-of-fit, measured by the Chi-square statistics, improved as the time from the initial measurement and number of thinnings increased. In all cases, the Chi-square goodness-of-fit tests indicated that the predicted diameter distributions were not different from the observed distributions at the 0.2573 significance level (for the poorest fit).

### *Predicted Stand Tables*

To evaluate the prediction system in terms of biological relationships, stand tables were generated for various combinations of ages, site indexes, and basal areas, all well within the ranges of the observed data. The numbers of trees per acre were estimated from stand age, site index, and basal area per acre. In all cases, the stands were assumed to have been previously thinned once. These stand tables are presented in Table 10.

### SIZE CLASS DISTRIBUTIONS

For a given site index and stand basal area, as age increases, the number of diameter classes also increases. This increase is always due to the addition of larger, not smaller, diameter classes. There is also a general decrease in the number of trees in the smaller diameter classes and a corresponding increase in the number of trees in the larger diameter classes. Finally, it should be noted that as age increases, total number of trees in the stand decreases, for a given site index.

For a given age and stand basal area, an increasing site index also tends to result in an increasing spread in the diameter distribution. Again, the increase in number of diameter classes is always due to the addition of larger diameter classes. With increasing site index there is also a decreasing number of trees in the smaller diameter classes and an increasing number in the larger classes. As was the case with age, a higher site index leads to a lower total number of trees for the stand at a given age.

For a given age and site index, effects due to varying levels of basal area are also present. An increase in basal area is followed by a slight increase in the number of diameter classes as well as an increase in the total number of trees.

In general, the stand tables demonstrate the expected biological relationships in terms of size class distributions due to factors such as age, site index, and stand density.

### VOLUME YIELDS

Total cubic-foot volume yields from the stand tables presented in Table 10 are summarized in Table 11. For a given site index and basal area, as age increases, so does volume, however, the rate of increase decreases with age. When age and site index are fixed, an increase in basal area results in an increase in total cubic-



foot volume which is fairly constant across the basal area classes. Higher volumes are also associated with higher site indexes. It should be noted that stands of higher site indexes have correspondingly larger volume differences between age periods than those of lower sites. The trends in total cubic-foot volume reflected in Table 11 are generally in agreement with known biological relationships.

### *Effect of Thinning Regime on Yield*

Six thinning regimes were outlined to determine the effects of thinning on volume yields and to answer the following questions:

1. How does the weight of thinning affect yield?
2. How does the number of thinnings affect yield?
3. How does the timing of thinnings affect yield?

### WEIGHT OF THINNING

To describe the influence of the weight of thinning on volume yields, two thinning regimes were specified, differing only in the amount of basal area removed at each thinning. Both regimes were modeled at three levels of site index to describe how the trends due to the thinning regimes are affected on "poor," "average," and "good" sites. The regimes are as follows:

#### Initial

conditions: Site index (base age 50) = 80, 110, 140 ft  
Initial age = 20 years  
Initial basal area = 80 sq ft/acre.

#### Regime 1: Thin to 50 sq ft/acre at age 20

Project to age 40 and thin to 70 sq ft/acre  
Project to age 50 and thin to 80 sq ft/acre  
Project to age 80.

#### Regime 2: Thin to 65 sq ft/acre at age 20

Project to age 40 and thin to 90 sq ft/acre  
Project to age 50 and thin to 110 sq ft/acre  
Project to age 80.

Stand-level summaries of total cubic-foot volume (ob) and board-foot volume yields per acre are given in Tables 12 and 13. Board-foot volume per acre is International 1/4-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob). In general, total cubic-foot and board-foot volume yields decrease as thinning weight increases. Due to the definition and structure of the thinning algorithm, for all three site indexes, the diameter distributions for the heavily thinned stands are shifted toward the larger diameter classes, as evidenced by the minimum, quadratic mean, and maximum diameters given for the final stand tables at age 80. The stand tables from regime 1 had less trees, basal area, total cubic-foot volume, and board-foot volume per acre. The differences in volume yields due to weight of thinning tend to increase with increasing site index.

### NUMBER OF THINNINGS

To demonstrate the effects of number of thinnings on volume yields, two additional thinning schedules were outlined. These regimes differ from regimes 1 and 2 only in that the stands are thinned once. Given the same initial conditions as before, including the three levels of site index, regimes 3 and 4 are as follows:

TABLE 10. Predicted stand tables for various combinations of age, site index, and basal area values (for stands thinned once).

SITE INDEX 90												
Basal area (sq ft/acre)												
90												
Age (years)	70				90				110			
	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/ acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/ acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/ acre)	Total cubic-foot volume
20	3	147.3	7.66	168.21	3	193.3	9.98	219.45	3	207.0	10.73	235.64
	4	178.0	15.48	345.27	4	217.7	18.91	422.20	4	245.2	21.35	476.17
	5	124.5	16.76	378.06	5	151.1	20.34	459.10	5	178.6	24.08	543.03
	6	69.1	13.34	303.40	6	86.1	16.65	378.92	6	106.9	20.70	470.35
	7	32.6	8.56	195.70	7	43.0	11.30	258.51	7	56.0	14.75	337.04
	8	13.5	4.63	106.31	8	19.3	6.62	152.24	8	26.5	9.09	208.62
	9	5.0	2.17	50.05	9	7.9	3.45	79.47	9	11.4	4.98	114.52
	10	1.7	0.90	20.82	10	3.0	1.62	37.43	10	4.6	2.46	56.74
	11	0.5	0.33	7.76	11	1.1	0.70	16.11	11	1.7	1.11	25.70
		Sum	572.1	69.83	1,575.58	Sum	722.4	89.56	2,023.43	Sum	838.6	109.71
30	3	0.6	0.03	0.65	3	1.2	0.07	1.24	3	1.4	0.08	1.48
	4	5.8	0.54	11.91	4	8.9	0.82	17.84	4	10.0	0.93	19.77
	5	17.5	2.46	60.70	5	23.6	3.30	80.18	5	25.6	3.59	85.84
	6	32.7	6.52	174.12	6	40.7	8.09	212.57	6	43.5	8.66	224.44
	7	44.4	11.94	338.09	7	52.7	14.16	394.41	7	56.8	15.25	419.19
	8	45.5	15.85	469.11	8	53.4	18.62	542.31	8	59.3	20.69	594.48
	9	34.8	15.27	468.02	9	42.4	18.61	561.27	9	50.0	21.98	653.75
	10	19.4	10.48	330.33	10	26.0	14.05	435.73	10	33.8	18.28	559.05
	11	7.7	4.97	160.32	11	12.1	7.89	250.33	11	18.1	11.80	369.29
	12	2.0	1.57	51.62	12	4.2	3.23	104.40	12	7.5	5.84	186.17
	Sum	210.4	69.64	2,064.84	Sum	266.1	88.78	2,631.17	Sum	308.9	109.86	3,203.74

[illegible]

**TABLE 10. Continued.**

SITE INDEX 110													
Basal area (sq ft/acre)													
70					90					110			
Age (years)	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	
20	3	32.7	1.79	40.20	3	47.5	2.58	57.44	3	52.8	2.86	63.38	
	4	82.1	7.32	176.30	4	104.7	9.31	222.38	4	114.9	10.21	242.46	
	5	96.9	13.25	334.82	5	116.4	15.88	398.47	5	129.1	17.64	439.90	
	6	81.2	15.84	414.17	6	96.2	18.77	487.02	6	110.1	21.51	554.77	
	7	52.8	13.94	373.63	7	64.4	17.03	452.97	7	77.4	20.50	541.74	
	8	27.5	9.47	258.64	8	36.1	12.46	337.45	8	46.4	16.01	430.92	
	9	11.7	5.10	141.28	9	17.3	7.54	207.34	9	24.1	10.53	287.43	
	10	4.1	2.21	61.95	10	7.2	3.85	107.05	10	11.0	5.93	163.79	
	11	1.2	0.78	22.06	11	2.6	1.68	47.05	11	4.4	2.90	80.78	
					12	0.8	0.63	17.77	12	1.6	1.24	34.80	
					13				13	0.5	0.47	13.19	
	Sum	390.3	69.71	1,823.05	Sum	493.2	89.72	2,334.96	Sum	572.4	109.78	2,853.16	
30	3	0.0	0.0	0.0	3	0.0	0.0	0.0	3	0.0	0.0	0.0	
	4	0.5	0.05	1.07	4	1.0	0.09	2.07	4	1.2	0.11	2.51	
	5	2.8	0.40	10.75	5	4.5	0.64	16.71	5	5.2	0.73	18.85	
	6	7.9	1.60	47.10	6	11.1	2.23	64.52	6	12.2	2.44	69.72	
	7	15.5	4.21	133.63	7	19.8	5.36	167.75	7	21.2	5.73	176.79	
	8	23.5	8.27	278.39	8	28.3	9.93	329.18	8	30.0	10.53	344.21	
	9	28.4	12.59	443.72	9	33.1	14.66	508.53	9	35.5	15.73	538.27	
	10	27.3	14.86	543.21	10	31.9	17.38	625.38	10	35.5	19.36	687.02	
	11	20.3	13.35	502.62	11	25.0	16.43	609.22	11	29.8	19.60	716.60	
	12	11.4	8.85	341.66	12	15.6	12.19	463.43	12	20.7	16.17	606.47	
	13	4.6	4.17	164.45	13	7.6	6.95	269.73	13	11.7	10.74	411.32	
	14	1.3	1.34	53.68	14	2.8	2.96	117.21	14	5.3	5.64	220.01	
					15	0.8	0.92	36.99	15	1.9	2.30	91.12	
									16	0.5	0.71	28.63	
		Sum	143.6	69.68	2,520.28	Sum	181.5	89.76	3,210.73	Sum	210.7	109.80	3,911.52

40	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5	0.1	0.02	0.44	0.44
	6	0.3	0.06	1.93	0.0	0.6	0.12	3.64	0.7	6	0.7	0.14	4.26	4.26
	7	1.2	0.33	10.97	1.9	1.9	0.53	17.45	2.2	7	2.2	0.59	19.30	19.30
	8	3.1	1.10	39.28	4.4	4.4	1.56	55.82	4.8	8	4.8	1.69	59.39	59.39
	9	6.2	2.76	105.04	8.1	8.1	3.61	137.12	8.5	9	8.5	3.80	142.15	142.15
	10	10.2	5.62	224.18	12.5	12.5	6.88	274.48	13.0	10	13.0	7.14	280.84	280.84
	11	14.2	9.40	390.26	16.6	16.6	11.02	457.70	17.3	11	17.3	11.47	469.50	469.50
	12	16.4	12.86	552.22	18.9	18.9	14.83	637.01	20.1	12	20.1	15.79	668.66	668.66
	13	15.3	14.08	622.21	18.0	18.0	16.56	731.94	20.1	13	20.1	18.54	807.59	807.59
	14	11.2	11.92	539.88	14.1	14.1	15.03	680.55	17.2	14	17.2	18.31	817.15	817.15
	15	6.1	7.46	345.13	8.8	8.8	10.77	498.27	12.2	15	12.2	14.92	680.40	680.40
	16	2.4	3.27	154.08	4.3	4.3	5.90	277.91	7.1	16	7.1	9.81	455.84	455.84
	17	0.6	0.94	45.13	1.5	1.5	2.37	113.64	3.2	17	3.2	5.07	239.35	239.35
	Sum	87.2	69.81	3,030.30	109.7	109.7	89.17	3,885.54	127.6	Sum	127.6	109.28	4,740.51	4,740.51
50	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5	0.0	0.0	0.0	0.0
	6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6	0.0	0.0	0.0	0.0
	7	0.0	0.0	0.0	0.2	0.2	0.05	1.84	0.2	7	0.2	0.07	2.17	2.17
	8	0.4	0.14	5.37	0.7	0.7	0.25	9.05	0.8	8	0.8	0.28	10.10	10.10
	9	1.1	0.51	20.43	1.7	1.7	0.77	30.40	1.9	9	1.9	0.83	32.50	32.50
	10	2.5	1.39	58.80	3.5	3.5	1.91	79.67	3.6	10	3.6	2.00	82.60	82.60
	11	4.7	3.10	137.65	6.0	6.0	3.96	173.30	6.1	11	6.1	4.08	176.03	176.03
	12	7.4	5.85	270.07	8.9	8.9	7.07	321.42	9.1	12	9.1	7.21	323.65	323.65
	13	10.1	9.36	446.44	11.8	11.8	10.89	511.89	12.1	13	12.1	11.19	518.77	518.77
	14	11.7	12.51	614.25	13.4	13.4	14.39	695.72	14.2	14	14.2	15.18	724.17	724.17
	15	11.1	13.63	685.48	13.1	13.1	16.03	794.10	14.5	15	14.5	17.85	872.28	872.28
	16	8.4	11.63	597.43	10.5	10.5	14.68	742.99	12.8	16	12.8	17.87	892.44	892.44
	17	4.7	7.40	387.26	6.8	6.8	10.71	552.23	9.5	17	9.5	14.92	759.26	759.26
	18	1.9	3.31	175.93	3.4	3.4	5.99	313.94	5.8	18	5.8	10.11	523.37	523.37
	Sum	64.0	68.82	3,399.11	81.3	81.3	89.14	4,357.15	94.5	Sum	94.5	109.19	5,317.48	5,317.48

TABLE 10. Continued.

SITE INDEX 130													
Basal area (sq ft/acre)													
70													
90													
110													
Age (years)	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	
20	3	4.7	0.27	5.88	3	7.9	0.44	9.69	3	9.4	0.53	11.36	
	4	22.3	2.03	50.57	4	31.5	2.86	70.22	4	35.3	3.20	77.81	
	5	43.1	5.98	161.82	5	54.7	7.58	202.70	5	59.8	8.27	219.14	
	6	56.3	11.11	318.96	6	67.2	13.25	375.91	6	73.2	14.44	405.71	
	7	55.4	14.78	442.78	7	64.6	17.24	510.56	7	71.8	19.18	562.49	
	8	42.3	14.67	454.23	8	50.4	17.49	535.29	8	58.6	20.35	616.58	
	9	25.3	11.03	350.37	9	32.3	14.14	443.89	9	40.3	17.66	548.77	
	10	11.7	6.29	203.96	10	17.1	9.19	294.54	10	23.5	12.68	402.19	
	11	4.2	2.71	89.31	11	7.4	4.82	157.18	11	11.6	7.58	244.57	
	12	1.1	0.87	29.22	12	2.6	2.04	67.55	12	4.9	3.79	123.88	
					13	0.8	0.70	23.36	13	1.7	1.58	52.35	
					14				14	0.5	0.55	18.46	
	Sum	266.3	69.75	2,107.10	Sum	336.5	89.76	2,690.90	Sum	390.5	109.79	3,283.32	
30	4	0.0	0.0	0.0	4	0.0	0.0	0.0	4	0.0	0.0	0.0	
	5	0.2	0.03	0.69	5	0.4	0.05	1.46	5	0.5	0.07	1.81	
	6	0.9	0.19	5.90	6	1.6	0.32	9.93	6	1.9	0.38	11.44	
	7	2.7	0.75	25.67	7	4.1	1.12	37.66	7	4.6	1.24	41.29	
	8	5.9	2.09	77.04	8	8.0	2.82	102.37	8	8.6	3.03	108.50	
	9	10.3	4.58	178.19	9	12.9	5.73	219.72	9	13.5	6.03	228.14	
	10	14.8	8.14	331.39	10	17.6	9.66	387.47	10	18.4	10.11	400.06	
	11	17.9	11.83	500.03	11	20.7	13.66	568.49	11	21.9	14.48	594.62	
	12	17.7	13.89	605.58	12	20.5	16.11	691.93	12	22.6	17.72	750.90	
	13	14.0	12.84	574.52	13	17.0	15.65	689.77	13	20.0	18.41	800.72	
	14	8.5	9.01	412.50	14	11.5	12.25	552.16	14	15.0	16.01	712.41	
	15	3.8	4.60	214.74	15	6.2	7.53	346.11	15	9.4	11.47	520.33	
	16	1.2	1.62	77.10	16	2.6	3.53	164.95	16	4.8	6.63	305.94	
					17	0.8	1.22	57.82	17	1.9	3.02	141.63	
					18				18	0.6	1.06	50.37	
		Sum	97.9	69.57	3,003.36	Sum	123.8	89.64	3,829.82	Sum	143.7	109.66	4,668.17

5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.1	0.03	1.08	0.1	0.04	1.33
8	0.2	0.08	3.20	0.4	0.15	5.81	0.5	0.18	6.69
9	0.7	0.31	13.10	1.1	0.50	20.62	1.2	0.56	22.61
10	1.6	0.90	39.75	2.3	1.29	56.36	2.5	1.38	59.57
11	3.2	2.11	97.65	4.2	2.79	127.58	4.4	2.91	131.29
12	5.3	4.19	202.13	6.6	5.21	247.49	6.8	5.33	250.51
13	7.7	7.16	357.24	9.2	8.48	416.79	9.4	8.65	420.25
14	9.8	10.46	536.96	11.2	12.04	609.23	11.6	12.45	622.08
15	10.4	12.83	674.74	12.0	14.75	765.30	12.9	15.78	808.83
16	9.2	12.82	689.07	11.0	15.31	811.74	12.5	17.45	913.90
17	6.4	10.04	550.36	8.4	13.12	709.56	10.5	16.54	883.27
18	3.3	5.88	327.67	5.1	9.02	495.94	7.5	13.17	715.60
19	1.2	2.43	137.33	2.5	4.79	267.42	4.4	8.60	474.39
Sum	59.2	69.21	3,629.23	75.0	89.36	4,641.67	87.0	109.32	5,662.39

[illegible]

**TABLE 11.** *Total cubic-foot volume yields for various combinations of site index, age, and basal area values of yellow-poplar stands thinned one time.*

Site index and age (years)	Basal area (sq ft/acre)		
	70	90	110
Site index 90	<i>cubic feet</i>		
20	1,576	2,023	2,479
30	2,065	2,631	3,204
40	2,471	3,151	3,837
50	2,760	3,516	4,315
Site index 110			
20	1,823	2,335	2,853
30	2,520	3,211	3,912
40	3,030	3,886	4,741
50	3,399	4,357	5,317
Site index 130			
20	2,107	2,691	3,283
30	3,003	3,830	4,668
40	3,629	4,642	5,662
50	4,043	5,196	6,346

Regime 3: Project to age 40 and thin to 70 sq ft/acre  
Project to age 80.

Regime 4: Project to age 40 and thin to 90 sq ft/acre  
Project to age 80.

Stand-level summaries of total cubic-foot volume (ob) and board-foot volume yields per acre are given in Tables 14 and 15. Board-foot volume per acre is International 1/4-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob). Upon comparison of yields from regimes 1 and 3, the additional thinnings in regime 1 resulted in increased cubic-foot and board-foot yields throughout the rotation at the low site. At the high site, regime 3 had the larger cubic-foot and board-foot volume yields. There were small differences in volume yields for the moderate sites. Similar trends are apparent when comparing yields from regimes 2 and 4. Because the coefficients for the basal area and cubic-foot volume projection equations in the "two-or-more" thinning case produce greater basal area and volume growth, these trends are as expected.

The faster growth rate associated with stands thinned two or more times has a greater effect at the low site index. For the low site index, the final stand tables showed the stand thinned more than once (regime 1) to have a diameter distribution with larger trees than the stand thinned only once. While it has fewer trees, the stand thinned three times has a higher basal area, cubic-foot volume, and board-foot volume. At the average site index, the stand tables from the two regimes are very similar in all respects. Finally at the high site index, the stand thinned only once has larger diameter trees, as well as greater numbers of trees, basal area, and cubic-foot and board-foot volumes. Similar trends were observed upon comparison of the stand tables from regimes 2 and 4.

#### TIMING OF THINNING

To illustrate the effect of timing of thinnings on volume yields, two thinning regimes were specified differing only in the time at which the thinnings occurred.



Given the same initial conditions and the three levels of site index, regimes 5 and 6 are given as:

Regime 5: Thin to 70 sq ft/acre at age 20  
Project to age 30 and thin to 80 sq ft/acre  
Project to age 40 and thin to 90 sq ft/acre  
Project to age 80.

Regime 6: Thin to 70 sq ft/acre at age 20  
Project to age 40 and thin to 80 sq ft/acre  
Project to age 50 and thin to 90 sq ft/acre  
Project to age 80.

Stand-level summaries of total cubic-foot volume (ob) and board-foot volume yields per acre, where again, board-foot volume per acre is International 1/4-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob), are given in Tables 16 and 17. The earlier thinnings of regime 5 resulted in greater cubic-foot and board-foot yields for the low and moderate site indexes. For the high site index, total cubic-foot and board-foot productions are similar for both the early and late thinnings. The differences in yields due to timing of thinnings tend to decrease as site index increases. For the low site index in particular, early thinnings result in substantial increases in both board-foot and cubic-foot yields.

Based on the final stand tables, the earlier thinnings of regime 5 resulted in greater numbers of trees, basal area, and cubic-foot and board-foot volumes per acre for all site indexes. In addition, the diameter distributions for the stands from regime 5 are shifted slightly toward larger diameter classes than those associated with the stands of regime 6 which were thinned at a later time. This trend becomes more pronounced as site index increases.

In general, as the weight of thinning increased, cubic-foot and board-foot volume yields decreased. The differences due to weight tended to be greater as site index increased. Additional thinnings resulted in greater volume yields, and as site index increased, the trends due to the number of thinnings reversed. Finally, early thinnings produced higher volume yields than the late thinnings—the differences in yields being smaller for the higher site index values. In the six thinning regimes, the differences in total cubic-foot and board-foot yields, as well as the corresponding basal areas and numbers of trees per acre, throughout the rotations were different due to changes in stand structures attributable to the weight, number, and timing of the thinnings.

In all of these comparisons, only the volume in specified size classes was considered; i.e., no consideration was given to the impact of thinning on the quality of the residual stand. When performing in-depth economic analyses of thinning alternatives, quality, as well as volume, relationships should be considered.

## DISCUSSION

### *Model Limitations and Recommendations*

Although the growth and yield model produced logical and consistent results, there are certain limitations in the prediction system. First, due to the structure of the data set, it was not possible to fit an equation to project basal area prior to the first thinning. At measurement periods 1 and 2, all stands were thinned. Thus no data were available on basal area growth in unthinned stands. Until such data become available, the stand level equation for basal area prediction after the first thinning can be used as the best approximation in such cases. Similarly, data



TABLE 13. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 2.

Site index and age (yrs.)	Before thinning				After thinning					Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)*
	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)				
Site index 80												
20	683	80	1,671	0	450	65	1,348	0	323	0	1,671	0
30	450	94	2,370	162							2,693	162
40	450	113	3,276	541	280	90	2,714	541	562	0	3,599	541
50	280	120	3,998	4,121	234	110	3,725	4,121	273	0	4,883	4,121
60	234	135	4,872	9,822							6,030	9,822
70	234	156	5,947	15,111							7,105	15,111
80	234	174	6,988	20,285							8,146	20,285
Site index 110												
20	431	80	2,098	56	280	65	1,725	56	373	0	2,098	56
30	280	108	3,753	3,434							4,126	3,434
40	280	139	5,620	10,061	126	90	3,876	10,061	1,744	0	5,993	10,061
50	126	120	5,723	20,958	107	110	5,307	20,305	416	653	7,840	20,958
60	107	135	7,028	29,744							9,561	30,397
70	107	156	8,524	38,245							11,057	38,898
80	107	174	9,903	46,036							12,436	46,689
Site index 140												
20	291	80	2,519	1,546	174	65	2,110	1,546	409	0	2,519	1,546
30	174	123	5,504	15,840							5,913	15,840
40	174	169	8,966	34,130	63	90	5,086	22,695	3,880	11,435	9,375	34,130
50	63	120	7,505	36,706	55	110	6,926	34,242	579	2,464	11,794	48,141
60	55	135	9,222	47,941							14,090	61,840
70	55	157	11,244	60,348							16,112	74,247
80	55	175	13,067	71,842							17,935	85,741

<sup>a</sup> Board-foot volume per acre; International 1/4-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 14. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 3.

Site index and age (yrs.)	Before thinning				After thinning				Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)*	
	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)				
Site index 80												
20	638	80	1,671	0						1,671	0	
30	420	108	2,804	285						2,804	285	
40	333	126	3,747	3,393	128	70	2,203	3,136	257	3,747	3,393	
50	128	86	3,015	7,304					1,544	4,559	7,561	
60	128	99	3,701	10,790						5,245	11,047	
70	128	109	4,354	14,112						5,898	14,369	
80	128	117	4,821	16,599						6,365	16,856	
Site index 110												
20	431	80	2,098	56						2,098	56	
30	306	124	4,224	6,610						4,224	6,610	
40	248	154	6,176	17,969	51	70	3,108	13,094	4,875	6,176	17,969	
50	51	93	4,710	22,086					3,068	7,778	26,961	
60	51	113	6,066	30,268						9,134	35,143	
70	51	130	7,393	38,081						10,461	42,956	
80	51	144	8,501	45,029						11,569	49,904	
Site index 140												
20	291	80	2,519	1,546						2,519	1,546	
30	221	141	6,029	19,030						6,029	19,030	
40	181	187	9,488	40,937	22	70	3,922	21,449	19,448	9,488	40,937	
50	22	101	6,630	38,854					5,566	12,196	58,342	
60	22	129	9,121	55,594						14,687	75,082	
70	22	154	11,449	71,733						17,015	91,221	
80	22	176	13,587	86,690						19,153	106,178	

\* Board-foot volume per acre. International  $\frac{1}{4}$ -inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 15. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 4.

Site index and age (yrs.)	Before thinning				After thinning				Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)*	
	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)				
Site index 80												
20	638	80	1,671	0						1,671	0	
30	420	108	2,804	285						2,804	285	
40	333	126	3,747	3,393	181	90	2,788	3,393	0	3,747	3,393	
50	181	105	3,659	7,180						4,618	7,180	
60	181	117	4,302	10,217						5,261	10,217	
70	181	126	4,838	12,875						5,797	12,875	
80	181	133	5,335	15,287						6,294	15,287	
Site index 110												
20	431	80	2,098	56						2,098	56	
30	306	124	4,224	6,610						4,224	6,610	
40	248	154	6,176	17,969	78	90	3,909	15,572	2,397	6,176	17,969	
50	78	114	5,635	24,774						7,902	27,171	
60	78	134	7,059	32,879						9,326	35,276	
70	78	150	8,369	40,490						10,636	42,887	
80	78	163	9,466	46,884						11,733	49,281	
Site index 140												
20	291	80	2,519	1,546						2,519	1,546	
30	221	141	6,029	19,030						6,029	19,030	
40	181	187	9,488	40,937	32	90	5,010	26,620	14,317	9,488	40,937	
50	32	124	8,059	45,520						12,537	59,837	
60	32	153	10,698	63,142						15,176	77,459	
70	32	178	13,133	79,378						17,611	93,695	
80	32	199	15,291	94,291						19,769	108,608	

<sup>a</sup> Board-foot volume per acre; International 1/4-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 16. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 5.

Site index and age (yrs.)	Before thinning				After thinning				Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac) <sup>a</sup>	
	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)				
Site index 80												
20	638	80	1,671	0	507	70	1,457	0	214	1,671	0	
30	507	99	2,464	150	322	80	2,054	150	410	2,678	150	
40	322	118	3,529	2,220	192	90	2,796	2,220	733	4,153	2,220	
50	192	120	4,143	9,136						5,500	9,136	
60	192	145	5,425	15,552						6,782	15,552	
70	192	166	6,541	21,358						7,898	21,358	
80	192	184	7,507	26,511						8,864	26,511	
Site index 110												
20	431	80	2,098	56	317	70	1,852	56	246	2,098	56	
30	317	113	3,893	3,091	160	80	2,897	3,091	996	4,139	3,091	
40	160	118	4,985	14,602	99	90	3,927	14,168	434	6,227	14,602	
50	99	120	5,813	23,910						8,113	24,344	
60	99	145	7,573	33,745						9,873	34,179	
70	99	166	9,178	42,801						11,478	43,235	
80	99	184	10,578	50,936						12,878	51,370	
Site index 140												
20	291	80	2,519	1,546	200	70	2,256	1,546	263	2,519	1,546	
30	200	129	5,692	14,977	87	80	3,729	13,795	1,963	5,955	14,977	
40	87	118	6,552	28,825	56	90	5,120	23,689	1,432	8,778	30,007	
50	56	120	7,511	38,042						11,229	44,360	
60	56	145	9,911	52,084						13,569	58,402	
70	56	166	11,936	64,595						15,594	70,913	
80	56	184	13,784	76,065						17,442	82,383	

<sup>a</sup> Board-foot volume per acre; International 1/4-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 17. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 6.

Site index and age (yrs.)	Before thinning				After thinning					Total vol. production (bd ft/ac)*	
	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Volume removed (cu ft/ac)		Volume removed (bd ft/ac)
Site index 80											
20	638	80	1,671	0	507	70	1,457	0	214	0	0
30	507	99	2,464	150							150
40	507	118	3,347	416	242	80	2,422	416	925	0	416
50	242	109	3,674	4,230	167	90	3,118	4,230	556	0	4,230
60	167	114	4,256	10,900							10,900
70	167	135	5,254	16,240							16,240
80	167	153	6,214	21,362							21,362
Site index 110											
20	431	80	2,098	56	317	70	1,852	56	246	0	56
30	317	113	3,893	3,091							3,091
40	317	144	5,705	8,854	113	80	3,440	8,854	2,265	0	8,854
50	113	109	5,219	19,273	81	90	4,398	17,673	821	1,600	19,273
60	81	114	5,980	26,301							26,301
70	81	135	7,535	35,095							35,095
80	81	154	8,849	42,870							42,870
Site index 140											
20	291	80	2,519	1,546	200	70	2,256	1,546	263	0	1,546
30	200	129	5,692	14,977							14,977
40	200	175	9,234	33,154	57	80	4,559	20,187	4,675	12,967	33,154
50	57	109	6,908	33,755	42	90	5,780	28,888	1,128	4,867	46,722
60	42	114	7,845	41,762							41,762
70	42	135	9,819	53,946							53,946
80	42	154	11,584	65,155							65,155
											71,780
											82,989

<sup>a</sup> Board-foot volume per acre; International 1/4-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

were available for stands thinned up to two times. For stands thinned more than twice, the equation for stands based on two thinnings was substituted.

Finally, there were no data on tree mortality. This represents a problem primarily for the unthinned stand table projections. Because of the thinnings made every five years, mortality was virtually nonexistent in the thinned stands. This may not be expected operationally, as repeated thinnings, as well as the thinning operations, can cause damage and death to the residual trees. However, based on the data used in this study, one can only assume no mortality when projecting the stands through time following thinnings. For unthinned stand projection, number of trees must be predicted from the projected age, site index, and basal area.

One recommended area for improvement in this study concerns the development of an appropriate stand-level growth and yield model. Using two sets of coefficients for the Sullivan and Clutter simultaneous growth and yield model—one for stands after one thinning and a second for stands after two thinnings, might suggest that the model form is an over-simplification of reality. The development of a generalized growth/growing stock theory that considers the changes in the relationships brought about by thinning in the population would represent a significant step forward in modeling methodology. While our procedures using two sets of coefficient estimates worked well, it should be pointed out that they indicate the need for a more generalized model, not a definitive solution to the problem.

Another possible refinement of the model is to redefine the basal area removal functions or the algorithm used to thin the stands. In most light to moderate thinnings no trees are removed from the larger diameter classes with the algorithm. However, in practice, larger trees are sometimes removed due to mortality, defect, etc. Also, this model is restricted to describing thinnings according to the removal patterns observed in the sample plots. Once data from stands thinned by other methods and diameter limit criteria become available, additional removal patterns could be formulated to simulate the various types of thinning, and thus increase the applicability and scope of this model. One method to obtain more realistic removal patterns for thinning, suggested by Cao and others (1982), is to establish stochastic models in which trees in each diameter class are assigned probabilities of being removed, and are cut or left depending on values of the random numbers generated.

### *Summary*

In this study a growth and yield model for thinned stands of yellow-poplar was developed. The model produces both stand-level and diameter distribution level estimates of number of trees, basal area, and cubic-foot volume per acre.

Development of the model consisted of two stages. In the first, equations to predict stand-level attributes were obtained. Then, in the second, stand tables were derived from the stand-level attributes by solving for the parameters of a three parameter Weibull distribution. The shape and scale parameters were obtained according to the parameter recovery procedure. The location parameter was estimated independently. When applying the system, the same stand-level basal area equation is used when deriving diameter distributions as when estimating overall stand basal area in order to ensure compatibility between the two levels of stand detail.

Overall, the parameter recovery procedure for estimating the parameters of the diameter distributions of the stands before thinnings gave reasonable estimates of number of trees, basal area, and cubic-foot volume per acre by diameter class. The thinning algorithm, which removed a proportion of basal area from each



class to simulate a thinning from below, produced stand and stock tables after thinning that were consistent with those generated before thinning, while adequately describing the observed diameter distributions after thinning. The growth and yield model for yellow-poplar provides detailed information about stand structure in an efficient manner that allows the evaluation of various thinning options.

## LITERATURE CITED

- AMATEIS, R. L., H. E. BURKHART, B. R. KNOEBEL, and P. T. SPRINZ. 1984. Yields and size class distributions for unthinned loblolly pine plantations and cutover site-prepared lands. VPI and SU, Sch For and Wildl Resour Publ FWS-2-84, 69 p.
- BAILEY, R. L. 1972. Development of unthinned stands of *Pinus radiata* in New Zealand. Unpublished Ph D diss, Univ Ga, Dep For, 67 p.
- BAILEY, R. L., and T. R. DELL. 1973. Quantifying diameter distributions with the Weibull function. *Forest Sci* 19:97-104.
- BAILEY, R. L., L. V. PIENAAR, B. D. SHIVER, and J. W. RHENEY. 1982. Stand structure and yield of site-prepared slash pine plantations. *The Univ Ga, Agric Exp Stn Bull* 291, 83 p.
- BECK, D. E. 1962. Yellow-poplar site index curves. *USDA Forest Serv Res Note* SE-180, 2 p. Southeast Forest Exp Stn.
- BECK, D. E. 1963. Cubic-foot volume tables for yellow-poplar in the southern Appalachians. *USDA Forest Serv Res Note* SE-16, 4 p. Southeast Forest Exp Stn.
- BECK, D. E. 1964. International 1/4-inch board-foot volumes and board-foot/cubic-foot ratios for southern Appalachian yellow-poplar. *USDA Forest Serv Res Note* SE-27, 4 p. Southeast Forest Exp Stn.
- BECK, D. E., and L. DELLA-BIANCA. 1970. Yield of unthinned yellow-poplar. *USDA Forest Serv Res Note* SE-58, 20 p. Southeast Forest Exp Stn.
- BECK, D. E., and L. DELLA-BIANCA. 1972. Growth and yield of thinned yellow-poplar. *USDA Forest Serv Res Pap* SE-101, 20 p. Southeast Forest Exp Stn.
- BECK, D. E., and L. DELLA-BIANCA. 1975. Board-foot and diameter growth of yellow-poplar after thinning. *USDA Forest Serv Res Pap* SE-123, 20 p. Southeast Forest Exp Stn.
- BENNETT, F. A. 1970. Variable density yield tables for managed stands of natural slash pine. *USDA Forest Serv Res Note* SE-141, 7 p. Southeast Forest Exp Stn.
- BENNETT, F. A., and J. L. CLUTTER. 1968. Multiple-product yield estimates for unthinned slash pine plantations—pulpwood, sawtimber, gum. *USDA Forest Serv Res Pap* SE-35, 21 p. Southeast Forest Exp Stn.
- BRENDER, E. V., and J. L. CLUTTER. 1970. Yield of even-aged, natural stands of loblolly pine. *Ga For Res Counc Rep* 23, 7 p.
- BUCKMAN, R. E. 1962. Growth and yield of red pine in Minnesota. *U S Dep Agric Tech Bull* 1272, 50 p.
- BURK, T. E., and H. E. BURKHART. 1984. Diameter distributions and yields of natural stands of loblolly pine. VPI and SU, Sch For and Wildl Resour Publ FWS-1-84, 46 p.
- BURKHART, H. E. 1977. Cubic-foot volume of loblolly pine to any merchantable top limit. *South J Appl For* 1:7-9.
- BURKHART, H. E., and P. T. SPRINZ. 1984. Compatible cubic volume and basal area projection equations for thinned old-field loblolly pine plantations. *Forest Sci* 30:86-93.
- BURKHART, H. E., and M. R. STRUB. 1974. A model for simulation of planted loblolly pine stands. *In* *Growth models for tree and stand simulation* (J. Fries, ed), p 128-135. Royal Coll For, Stockholm, Sweden.
- BURKHART, H. E., R. C. PARKER, and R. G. ODERWALD. 1972a. Yields for natural stands of loblolly pine. VPI and SU, Div For and Wildl Resour Publ FWS-2-72, 63 p.
- BURKHART, H. E., R. C. PARKER, M. R. STRUB, and R. G. ODERWALD. 1972b. Yields of old-field loblolly pine plantations. VPI and SU, Div For and Wildl Resour Publ FWS-3-72, 51 p.
- CAO, Q. V., and H. E. BURKHART. 1980. Cubic-foot volume of loblolly pine to any height limit. *South J Appl For* 4:166-168.
- CAO, Q. V., and H. E. BURKHART. 1984. A segmented distribution approach for modeling diameter frequency data. *Forest Sci* 30:129-137.
- CAO, Q. V., H. E. BURKHART, and R. C. LEMIN, JR. 1982. Diameter distributions and yields of thinned loblolly pine plantations. VPI and SU, Sch For and Wildl Resour Publ FWS-1-82, 62 p.

- CLUTTER, J. L. 1963. Compatible growth and yield models for loblolly pine. *Forest Sci* 9:354-371.
- CLUTTER, J. L., and F. A. BENNETT. 1965. Diameter distributions in old-field slash pine plantations. *Ga For Res Counc Rep* 13, 9 p.
- COILE, T. S., and F. X. SCHUMACHER. 1964. Soil-site relations, stand structure and yields of slash and loblolly pine plantations in the southern United States. T. S. Coile, Inc., Durham, N.C. 296 p.
- DANIELS, R. F., H. E. BURKHART, and M. R. STRUB. 1979. Yield estimates for loblolly pine plantations. *J For* 77:581-583, 586.
- DELL, T. R., D. P. FEDUCCIA, T. E. CAMPBELL, W. F. MANN, JR., and B. H. POLMER. 1979. Yields of unthinned slash pine plantations on cutover sites in the west gulf region. USDA Forest Serv Res Pap SO-147, 84 p.
- FEDUCCIA, D. P., T. R. DELL, W. F. MANN, JR., T. E. CAMPBELL, and B. H. POLMER. 1979. Yields of unthinned loblolly pine plantations on cutover sites in the west gulf region. USDA Forest Serv Res Pap SO-148, 87 p.
- FRAZIER, J. R. 1981. Compatible whole-stand and diameter distribution models for loblolly pine stands. Unpublished Ph D diss, VPI and SU, Dep For. 125 p.
- GOEBEL, N. B., and J. R. WARNER. 1969. Volume yields of loblolly pine plantations for a variety of sites in the South Carolina Piedmont. *S C Agric Exp Stn For Res Ser* 13, 15 p.
- HAFLEY, W. L., W. D. SMITH, and M. A. BUFORD. 1982. A new yield prediction model for unthinned loblolly pine plantations. North Carolina State Univ, Sch Forest Resour, South For Res Cent, Bioecon Modeling Proj, Tech Rep 1, 65 p. Raleigh, N.C.
- HYINK, D. M. 1980. Diameter distribution approaches to growth and yield modeling. *In* Forecasting forest stand dynamics (K. M. Brown and F. R. Clarke, eds), p 138-163. Lakehead Univ, Sch For, Thunderbay, Ontario.
- HYINK, D. M., and J. W. MOSER, JR. 1983. A generalized framework for projecting forest yield and stand structure using diameter distributions. *Forest Sci* 29:85-95.
- KNOEBEL, B. R., H. E. BURKHART, and D. E. BECK. 1984. Stem volume and taper functions for yellow-poplar in the southern Appalachians. *South J Appl For* 8:185-188.
- LENHART, J. D. 1972. Cubic volume yields for unthinned old-field loblolly pine plantations in the interior west gulf coastal plain. *Texas For Pap* 14, 46 p.
- LENHART, J. D., and J. L. CLUTTER. 1971. Cubic-foot yield tables for old-field loblolly pine plantations in the Georgia Piedmont. *Ga For Res Counc Rep* 22—Ser 3, 12 p.
- LOHREY, R. E., and R. L. BAILEY. 1976. Yield tables and stand structure for unthinned longleaf pine plantations in Louisiana and Texas. USDA Forest Serv Res Pap SO-133, 53 p. South Forest Exp Stn.
- MACKINNEY, A. L., and L. E. CHAIKEN. 1939. Volume, yield, and growth of loblolly pine in the Mid-Atlantic Coastal Region. USDA Forest Serv Tech Note 33, 30 p.
- MACKINNEY, A. L., F. X. SCHUMACHER, and L. E. CHAIKEN. 1937. Construction of yield tables for non-normal loblolly pine stands. *J Agric Res* 54:531-545.
- MATNEY, T. G., and A. D. SULLIVAN. 1982. Compatible stand and stock tables for thinned and unthinned loblolly pine stands. *Forest Sci* 28:161-171.
- MCCARTHY, E. F. 1933. Yellow-poplar characteristics, growth, and management. *U S Dep Agric Tech Bull* 356, 57 p.
- MCGEE, C. E., and L. DELLA-BIANCA. 1967. Diameter distributions in natural yellow-poplar stands. USDA Forest Serv Res Pap SE-25, 7 p. Southeast Forest Exp Stn.
- MENDENHALL, W., and R. L. SCHEAFFER. 1973. Mathematical statistics with applications. Duxbury Press. 561 p.
- MURPHY, P. A., and R. C. BELTZ. 1981. Growth and yield of shortleaf pine in the west gulf. USDA Forest Serv Res Pap SO-169, 15 p.
- MURPHY, P. A., and H. S. STERNITZKE. 1979. Growth and yield estimation for loblolly pine in the west gulf. USDA Forest Serv Res Pap SO-154, 8 p.
- SCHREUDER, H. T., and W. T. SWANK. 1974. Coniferous stands characterized with the Weibull distribution. *Can J Forest Res* 4:518-523.
- SCHUMACHER, F. X., and T. S. COILE. 1960. Growth and yield of natural stands of the southern pines. T. S. Coile, Inc., Durham, N.C. 115 p.
- SMALLEY, G. W., and R. L. BAILEY. 1974a. Yield tables and stand structure for loblolly pine plantations in Tennessee, Alabama and Georgia highlands. USDA Forest Serv Res Pap SO-96, 81 p. South Forest Exp Stn.
- SMALLEY, G. W., and R. L. BAILEY. 1974b. Yield tables and stand structure for shortleaf pine

- plantations in Tennessee, Alabama and Georgia highlands. USDA Forest Serv Res Pap SO-97, 57 p.
- STRUB, M. R., and H. E. BURKHART. 1975. A class-interval-free method for obtaining expected yields from diameter distributions. *Forest Sci* 21:67-69.
- SULLIVAN, A. D., and J. L. CUTTER. 1972. A simultaneous growth and yield model for loblolly pine. *Forest Sci* 18:76-86.
- SULLIVAN, A. D., and H. L. WILLISTON. 1977. Growth and yield of thinned loblolly pine plantations in loessial soil areas. *Miss Agric For Exp Stn Tech Bull* 86, 16 p.

## APPENDICES

### *Appendix 1. Example Run of Yellow-Poplar Growth and Yield Program*

A GROWTH AND YIELD PREDICTION MODEL FOR THINNED STANDS  
OF YELLOW-POPLAR.

A RESPONSE CAN BE ENTERED AS EITHER INTEGER- OR REAL-VALUED.

YOU MAY ENTER : "9999" AT ANY TIME TO TERMINATE THE PROGRAM.

"8888" AT ANY TIME TO RESTART THE PROGRAM.

ENTER AGE AT BEGINNING OF PROJECTION PERIOD.

20.

ENTER AGE AT END OF PROJECTION PERIOD.

20.

ENTER SITE INDEX (BASE AGE 50).

100.

EITHER NUMBER OF TREES OR BASAL AREA  
PER ACRE MUST BE KNOWN.

ENTER BASAL AREA PER ACRE AT BEGINNING  
OF PROJECTION PERIOD IF KNOWN,  
OTHERWISE ENTER 0.

80.

SPECIFY NUMBER OF TREES PER ACRE IF KNOWN  
OTHERWISE ENTER 0.

0.

ENTER NUMBER OF PREVIOUS THINNINGS AS:

- 0 IF STAND HAS NOT BEEN PREVIOUSLY THINNED,
- 1 IF STAND HAS BEEN PREVIOUSLY THINNED ONCE,
- 2 IF STAND HAS BEEN PREVIOUSLY THINNED MORE THAN ONCE.

0.

ENTER 1 FOR WHOLE STAND GROWTH AND YIELD ESTIMATES,  
OR 2 FOR DIAMETER DISTRIBUTION LEVEL ESTIMATES.

2.

# PREDICTED STAND/STOCK TABLE

DBH (IN)	TREES (/AC)	BASAL AREA (SQ FT/AC)	TOTAL HEIGHT (FEET)	TOTAL CUBIC-FOOT VOLUME (OB)	INTERNATIONAL 1/4 BOARD-FOOT VOLUME 11 IN+, 8-IN OB TOP
3	33.7	1.9	50	42	0
4	110.5	9.9	53	233	0
5	143.1	19.6	55	473	0
6	114.0	22.2	56	546	0
7	60.8	15.9	57	399	0
8	22.2	7.6	58	191	0
9	5.5	2.4	58	61	0
10	1.1	0.6	58	15	0
TOTAL	490.9	80.0	---	1961	0

## STAND TABLE SUMMARY

### INPUT SUMMARY :

INITIAL AGE = 20  
 PROJECTED AGE = 20  
 SITE INDEX (FT, BASE AGE 50 FT) = 100  
 INITIAL BASAL AREA (SQ FT/AC) = 80  
 NUMBER OF TREES (/AC) = 0  
 NUMBER OF  
 PREVIOUS THINNINGS = 0

### PROJECTION SUMMARY :

BASAL AREA (SQ FT/AC) = 80  
 NUMBER OF TREES (/AC) = 491  
 MINIMUM DIAMETER (IN) = 3.0  
 QUADRATIC MEAN DIAMETER (IN) = 5.5  
 MAXIMUM DIAMETER (IN) = 10.0  
 AVERAGE HEIGHT OF DOMINANTS  
 AND CODOMINANTS (FT) = 53  
 CUBIC-FOOT VOLUME = 1961  
 BOARD-FOOT VOLUME = 0  
 (11 IN+ TO AN 8-IN OB TOP)

DO YOU WANT TO THIN THE STAND AT THIS TIME?

ENTER 1 FOR YES  
 0 FOR NO

1.

SPECIFY THE RESIDUAL BASAL AREA DESIRED

50.

## STAND/STOCK TABLE AFTER THINNING

DBH (IN)	TREES (/AC)	BASAL AREA (SQ FT/AC)	TOTAL HEIGHT (FEET)	TOTAL CUBIC-FOOT VOLUME (OB)	INTERNATIONAL 1/4 BOARD-FOOT VOLUME 11 IN+, 8-IN OB TOP
3	2.7	0.1	50	3	0
4	19.0	1.9	53	40	0
5	56.4	7.7	55	186	0
6	72.4	14.0	56	347	0
7	59.7	15.7	57	392	0
8	22.2	7.6	58	191	0
9	5.5	2.4	58	61	0
10	1.1	0.6	59	15	0
TOTAL	239.0	50.0	---	1236	0

# STAND TABLE SUMMARY AFTER THINNING

AGE = 20  
 SITE INDEX (FT, BASE AGE 50 FT) = 100  
 MINIMUM DIAMETER (IN) = 3.0  
 QUADRATIC MEAN DIAMETER (IN) = 6.2  
 MAXIMUM DIAMETER (IN) = 10.0

BASAL AREA (SQ FT/AC) = 50  
 CUBIC-FOOT VOLUME = 1236  
 BOARD-FOOT VOLUME = 0  
 (11. IN+ TO AN 8-IN DB TOP)

NUMBER OF TREES (/AC) PRIOR TO THINNING = 491  
 NUMBER OF TREES (/AC) REMOVED IN THINNING = 252

BASAL AREA (SQ FT/AC) PRIOR TO THINNING = 80  
 BASAL AREA (SQ FT/AC) REMOVED IN THINNING = 30

CUBIC-FOOT VOLUME PRIOR TO THINNING = 1961  
 CUBIC-FOOT VOLUME REMOVED IN THINNING = 724

BOARD-FOOT VOLUME PRIOR TO THINNING = 0  
 BOARD-FOOT VOLUME REMOVED IN THINNING = 0

DO YOU WANT TO RETHIN THE PREDICTED STAND TABLE  
 TO ANOTHER LEVEL OF RESIDUAL BASAL AREA ?

ENTER 1 FOR YES  
 0 FOR NO

0.

DO YOU WANT TO MAKE ANOTHER PROJECTION ?

ENTER 1 FOR YES  
 0 FOR NO

1.

ENTER AGE AT BEGINNING OF PROJECTION PERIOD.

20.

ENTER AGE AT END OF PROJECTION PERIOD.

40.

ENTER SITE INDEX (BASE AGE 50).

100.

EITHER NUMBER OF TREES OR BASAL AREA  
 PER ACRE MUST BE KNOWN.

ENTER BASAL AREA PER ACRE AT BEGINNING  
 OF PROJECTION PERIOD IF KNOWN,  
 OTHERWISE ENTER 0.

50.

FOR PROJECTION OF STANDS,  
ENTER THE KNOWN NUMBER OF TREES OR THE NUMBER OF TREES  
OBTAINED FROM A PREVIOUSLY PREDICTED STAND TABLE IF POSSIBLE,  
OTHERWISE ENTER 0.0.

239.

ENTER NUMBER OF PREVIOUS THINNINGS AS:

- 0 IF STAND HAS NOT BEEN PREVIOUSLY THINNED,
- 1 IF STAND HAS BEEN PREVIOUSLY THINNED ONCE,
- 2 IF STAND HAS BEEN PREVIOUSLY THINNED MORE THAN ONCE.

1.

ENTER 1 FOR WHOLE STAND GROWTH AND YIELD ESTIMATES,  
OR 2 FOR DIAMETER DISTRIBUTION LEVEL ESTIMATES.

1.

#### WHOLE STAND GROWTH AND YIELD ESTIMATES

INITIAL AGE = 20.	SITE INDEX (BASE AGE 50) = 100.
PROJECTED AGE = 40.	NUMBER OF PREVIOUS THINNINGS = 1.
INITIAL BASAL AREA = 50.0	CUBIC-FOOT VOLUME = 4431.1
PROJECTED BASAL AREA = 113.5	BOARD-FOOT VOLUME = 11982.2

DO YOU WANT THE CORRESPONDING STAND TABLE ?

ENTER 1 FOR YES  
0 FOR NO

1.

#### PREDICTED STAND/STOCK TABLE

DBH (IN)	TREES (/AC)	BASAL AREA (SQ FT/AC)	TOTAL HEIGHT (FEET)	TOTAL CUBIC-FOOT VOLUME (OB)	INTERNATIONAL 1/4 BOARD-FOOT VOLUME 11 IN+, 8-IN OB TOP
4	2.2	0.2	47	5	0
5	8.7	1.2	57	32	0
6	18.6	3.7	66	109	0
7	29.7	8.0	72	256	0
8	38.4	13.5	78	463	0
9	41.6	18.4	83	668	0
10	37.8	20.6	86	782	0
11	28.8	18.9	90	746	2184
12	18.2	14.2	93	576	1886
13	9.4	8.6	95	358	1277
14	3.9	4.1	97	177	676
15	1.8	2.2	97	94	380
-----					
TOTAL	239.0	113.5	---	4266	6403

# STAND TABLE SUMMARY

-----

## INPUT SUMMARY :

INITIAL AGE = 20  
 PROJECTED AGE = 40  
 SITE INDEX (FT, BASE AGE 50 FT) = 100  
 INITIAL BASAL AREA (SQ FT/AC) = 50  
 NUMBER OF TREES (/AC) = 239  
 NUMBER OF  
 PREVIOUS THINNINGS = 1

## PROJECTION SUMMARY :

BASAL AREA (SQ FT/AC) = 114  
 NUMBER OF TREES (/AC) = 239  
 MINIMUM DIAMETER (IN) = 4.0  
 QUADRATIC MEAN DIAMETER (IN) = 9.3  
 MAXIMUM DIAMETER (IN) = 15.0  
 AVERAGE HEIGHT OF DOMINANTS  
 AND CODOMINANTS (FT) = 90  
 CUBIC-FOOT VOLUME = 4266  
 BOARD-FOOT VOLUME = 6403  
 (11 IN+ TO AN 8-IN OB TOP)

DO YOU WANT TO THIN THE STAND AT THIS TIME?

ENTER 1 FOR YES  
 0 FOR NO

1.

SPECIFY THE RESIDUAL BASAL AREA DESIRED

70.

## STAND/STOCK TABLE AFTER THINNING

DBH (IN)	TREES (/AC)	BASAL AREA (SQ FT/AC)	TOTAL HEIGHT (FEET)	TOTAL CUBIC-FOOT VOLUME (OB)	INTERNATIONAL 1/4 BOARD-FOOT VOLUME 11 IN+, 8-IN OB TOP
4	0.0	0.0	0	0	0
5	0.0	0.0	0	0	0
6	0.0	0.0	0	0	0
7	0.0	0.0	0	0	0
8	0.0	0.0	0	0	0
9	4.8	2.1	83	77	0
10	36.6	19.9	86	757	0
11	28.8	18.9	90	746	2184
12	18.2	14.2	93	576	1886
13	9.4	8.6	95	358	1277
14	3.9	4.1	97	177	676
15	1.8	2.2	99	94	380
TOTAL	103.4	70.0	---	2786	6403



# STAND TABLE SUMMARY AFTER THINNING

AGE = 40  
 SITE INDEX (FT, BASE AGE 50 FT) = 100  
 MINIMUM DIAMETER (IN) = 9.0  
 QUADRATIC MEAN DIAMETER (IN) = 11.1  
 MAXIMUM DIAMETER (IN) = 15.0

BASAL AREA (SQ FT/AC) = 70  
 CUBIC-FOOT VOLUME = 2786  
 BOARD-FOOT VOLUME = 6403  
 (11 IN+ TO AN 8-IN OB TOP)

NUMBER OF TREES (/AC) PRIOR TO THINNING = 239  
 NUMBER OF TREES (/AC) REMOVED IN THINNING = 136

BASAL AREA (SQ FT/AC) PRIOR TO THINNING = 114  
 BASAL AREA (SQ FT/AC) REMOVED IN THINNING = 44

CUBIC-FOOT VOLUME PRIOR TO THINNING = 4266  
 CUBIC-FOOT VOLUME REMOVED IN THINNING = 1480

BOARD-FOOT VOLUME PRIOR TO THINNING = 6403  
 BOARD-FOOT VOLUME REMOVED IN THINNING = 0

DO YOU WANT TO RETHIN THE PREDICTED STAND TABLE  
 TO ANOTHER LEVEL OF RESIDUAL BASAL AREA ?

ENTER 1 FOR YES  
 0 FOR NO

1.

THE RESIDUAL BASAL AREA PREVIOUSLY SPECIFIED WAS 70.00

SPECIFY THE RESIDUAL BASAL AREA DESIRED

80.

## STAND/STOCK TABLE AFTER THINNING

DBH (IN)	TREES (/AC)	BASAL AREA (SQ FT/AC)	TOTAL HEIGHT (FEET)	TOTAL CUBIC-FOOT VOLUME (OB)	INTERNATIONAL 1/4 BOARD-FOOT VOLUME 11 IN+, 8-IN OB TOP
4	0.0	0.0	0	0	0
5	0.0	0.0	0	0	0
6	0.0	0.0	0	0	0
7	0.0	0.0	0	0	0
8	0.0	0.0	0	0	0
9	27.4	12.1	83	441	0
10	36.6	19.9	86	757	0
11	28.8	18.9	90	746	2184
12	18.2	14.2	93	576	1886
13	9.4	8.6	95	358	1277
14	3.9	4.1	97	177	676
15	1.8	2.2	99	94	380
<hr/>					
TOTAL	126.1	80.0	---	3150	6403

STAND TABLE SUMMARY AFTER THINNING

AGE = 40  
SITE INDEX (FT, BASE AGE 50 FT) = 100  
MINIMUM DIAMETER (IN) = 9.0  
QUADRATIC MEAN DIAMETER (IN) = 10.8  
MAXIMUM DIAMETER (IN) = 15.0

BASAL AREA (SQ FT/AC) = 80  
CUBIC-FOOT VOLUME = 3150  
BOARD-FOOT VOLUME = 6403  
(11 IN+ TO AN 8-IN DB TOP)

NUMBER OF TREES (/AC) PRIOR TO THINNING = 239  
NUMBER OF TREES (/AC) REMOVED IN THINNING = 113

BASAL AREA (SQ FT/AC) PRIOR TO THINNING = 114  
BASAL AREA (SQ FT/AC) REMOVED IN THINNING = 34

CUBIC-FOOT VOLUME PRIOR TO THINNING = 4266  
CUBIC-FOOT VOLUME REMOVED IN THINNING = 1116

BOARD-FOOT VOLUME PRIOR TO THINNING = 6403  
BOARD-FOOT VOLUME REMOVED IN THINNING = 0

DO YOU WANT TO RETHIN THE PREDICTED STAND TABLE  
TO ANOTHER LEVEL OF RESIDUAL BASAL AREA ?

ENTER 1 FOR YES  
0 FOR NO

0.

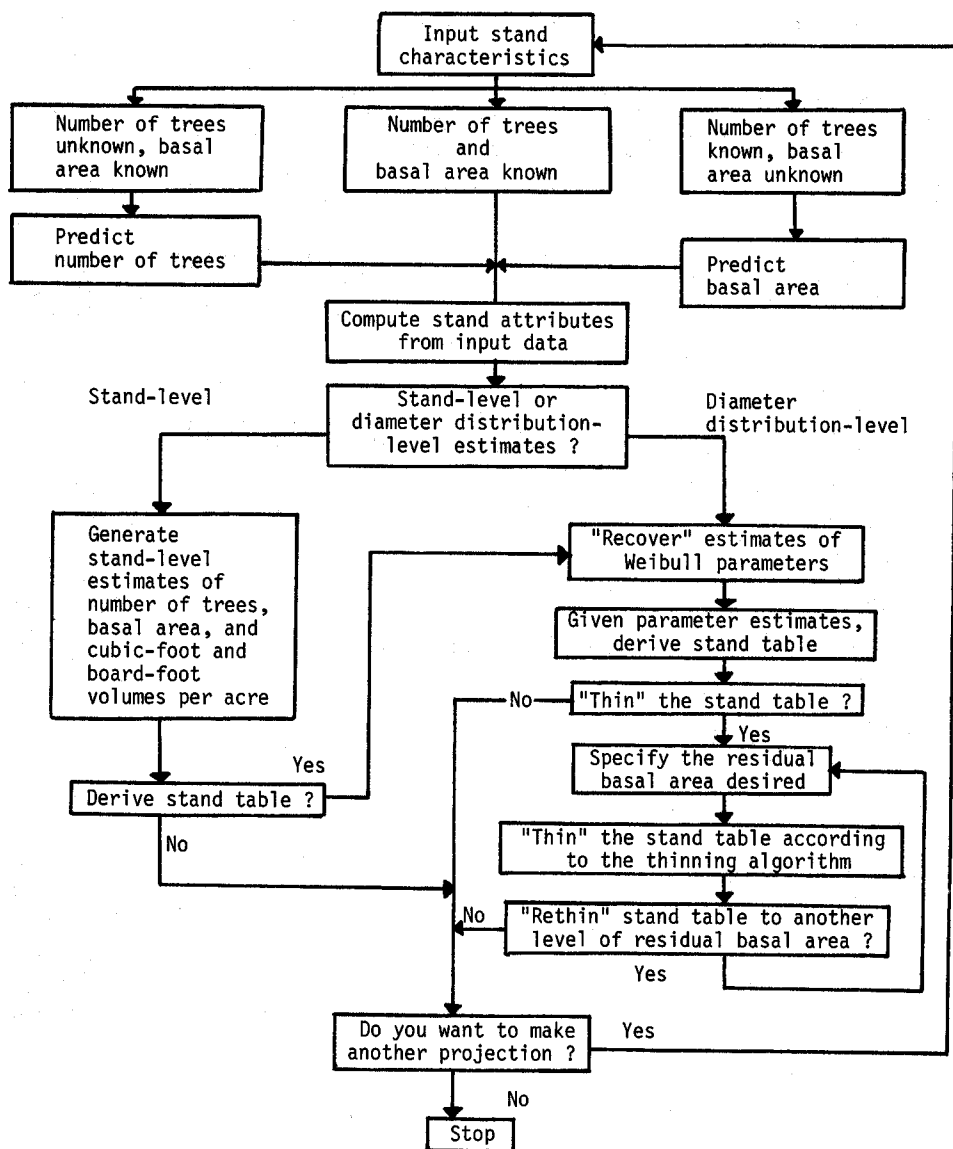
DO YOU WANT TO MAKE ANOTHER PROJECTION ?

ENTER 1 FOR YES  
0 FOR NO

0.

THE PROGRAM HAS BEEN TERMINATED BY THE USER

Appendix 2. Flow Chart Diagram of Yellow-Poplar Growth and Yield Program



### Appendix 3. Source Code for Yellow-Poplar Growth and Yield Program

C		YP000010
C	A GROWTH AND YIELD PREDICTION MODEL FOR THINNED STANDS	YP000020
C	OF YELLOW-POPLAR.	YP000030
C		YP000040
C		YP000050
C	MOMENT-BASED THREE PARAMETER WEIBULL SYSTEM WITH CONSTANT 'A'.	YP000060
C	USES PREDICTED AVERAGE DIAMETER AND BASAL AREA TO OBTAIN	YP000070
C	ESTIMATES OF THE WEIBULL PARAMETERS IN ORDER TO GENERATE STAND	YP000080
C	TABLES FOR THINNED STANDS OF YELLOW-POPLAR.	YP000090
C	-----	YP000100
C		YP000110
C		YP000120
C		YP000130
C		YP000140
C	QUESTIONS, RESPONSES, AND OUTPUT ARE SENT TO FILE 10 (TERMINAL)	YP000150
C	QUESTIONS, RESPONSES, AND OUTPUT ARE SENT TO FILE 4 (DISK)	YP000160
C	OUTPUT ONLY IS SENT TO FILE 11 (DISK)	YP000170
C		YP000180
C		YP000190
C		YP000200
C		YP000210
C	IMPLICIT REAL*8 (Z)	YP000220
C	REAL*4 NT,NT1,NT11	YP000230
C	DIMENSION DCL(50),BCL(50),VCL(50),ODCL(50),BAREM(50),TREM(50),	YP000240
C	:RBA(50),RNT(50),PROP(50),BAR(50),RNTR(50),OBCL(50),BVCL(50)	YP000250
C	COMMON/AREA1/DAVG,D2AVG,A,B,C	YP000260
C	COMMON/AREA2/HTCON,DMAX,HDOM	YP000270
C	EXTERNAL TREEHT,ZFCV,VDIST,BDIST	YP000280
C		YP000290
C	WRITE(10,100)	YP000300
C	WRITE(4,100)	YP000310
C	100 FORMAT(/5X,	YP000320
C	: 'A GROWTH AND YIELD PREDICTION MODEL FOR THINNED STANDS' /5X,	YP000330
C	: 'OF YELLOW-POPLAR.' /5X,	YP000340
C	: 'A RESPONSE CAN BE ENTERED AS EITHER INTEGER- OR REAL-VALUED.' /5X,	YP000350
C	: 5X, 'YOU MAY ENTER : "9999" AT ANY TIME TO TERMINATE THE PROGRAM.' /5X,	YP000360
C	: //, 21X, "8888" AT ANY TIME TO RESTART THE PROGRAM.' //)	YP000370
C		YP000380
C		YP000390
C		YP000400
C	110 DO 120 KLM=1,50	YP000410
C	BAR(KLM)=0.0	YP000420
C	RNTR(KLM)=0.0	YP000430
C	OBCL(KLM)=0.0	YP000440
C	ODCL(KLM)=0.0	YP000450
C	BCL(KLM)=0.0	YP000460
C	DCL(KLM)=0.0	YP000470
C	VCL(KLM)=0.0	YP000480
C	BAREM(KLM)=0.0	YP000490
C	TREM(KLM)=0.0	YP000500
C	120 CONTINUE	YP000510
C		YP000520
C		YP000530
C	-----	YP000540
C	*** INPUT DATA	YP000550
C		YP000560
C	-----	YP000570
C		YP000580
C		YP000590
C		YP000600
C	130 WRITE(10,140)	YP000610
C	WRITE(4,140)	YP000620
C	140 FORMAT(/5X, 'ENTER AGE AT BEGINNING OF PROJECTION PERIOD.')	YP000630
C	READ(5,*)AGE1	YP000640
C	WRITE(4,333)AGE1	YP000650
C	333 FORMAT(/5X,F5.0/)	YP000660
C	IF(AGE1.LE.90.AND.AGE1.GE.15)GO TO 150	YP000670
C	IF(AGE1.EQ.9999) GO TO 730	YP000680
C	IF(AGE1.EQ.8888)GO TO 710	YP000690
C	CALL RANGE(AGE1,1,RESP)	YP000700
C	IF(RESP.EQ.1)GO TO 130	YP000710
C	IF(RESP.EQ.0)GO TO 150	YP000720
C	IF(RESP.EQ.9999 )GO TO 730	YP000730
C	IF(RESP.EQ.8888)GO TO 710	YP000740

150	WRITE(10,160)	YP000750
	WRITE(4,160)	YP000760
160	FORMAT(/5X,'ENTER AGE AT END OF PROJECTION PERIOD.'//)	YP000770
170	CONTINUE	YP000780
	READ(5,*)AGE2	YP000790
	WRITE(4,333)AGE2	YP000800
	IF(AGE2.EQ.9999) GO TO 730	YP000810
	IF(AGE2.EQ.8888)GO TO 710	YP000820
	IF(AGE2.LT.AGE1)WRITE(10,180)AGE2,AGE1	YP000830
	IF(AGE2.LT.AGE1)WRITE(4,180)AGE2,AGE1	YP000840
180	FORMAT(/5X,'PROJECTED AGE OF',F5.0,1X,	YP000850
:	'IS LESS THAN INITIAL AGE OF',F5.0,	YP000860
:	/5X,'YOU MUST REENTER AGE AT END OF PROJECTION PERIOD '/')	YP000870
	IF(AGE2.LT.AGE1)GO TO 170	YP000880
	IF(AGE2.LT.90.AND.AGE2.GT.15)GO TO 190	YP000890
	CALL RANGE(AGE2,1,RESP)	YP000900
	IF(RESP.EQ.1)GO TO 150	YP000910
	IF(RESP.EQ.0)GO TO 190	YP000920
	IF(RESP.EQ.9999.0)GO TO 730	YP000930
	IF(RESP.EQ.8888)GO TO 710	YP000940
190	WRITE(10,200)	YP000950
	WRITE(4,200)	YP000960
200	FORMAT(/5X,'ENTER SITE INDEX (BASE AGE 50).')	YP000970
	READ(5,*)SITE	YP000980
	WRITE(4,333)SITE	YP000990
	IF(SITE.GT.75.AND.SITE.LT.140)GO TO 210	YP001000
	IF(SITE.EQ.9999) GO TO 730	YP001010
	IF(SITE.EQ.8888)GO TO 710	YP001020
	CALL RANGE(SITE,2,RESP)	YP001030
	IF(RESP.EQ.1)GO TO 190	YP001040
	IF(RESP.EQ.0)GO TO 210	YP001050
	IF(RESP.EQ.9999)GO TO 730	YP001060
	IF(RESP.EQ.8888)GO TO 710	YP001070
210	WRITE(10,220)	YP001080
	WRITE(4,220)	YP001090
220	FORMAT(/5X,'EITHER NUMBER OF TREES OR BASAL AREA'	YP001100
:	/5X,'PER ACRE MUST BE KNOWN.'//)	YP001110
	WRITE(10,230)	YP001120
	WRITE(4,230)	YP001130
230	FORMAT(5X,'ENTER BASAL AREA PER ACRE AT BEGINNING'/	YP001140
:	:11X,'OF PROJECTION PERIOD IF KNOWN,'/11X,'OTHERWISE ENTER 0.')	YP001150
	READ(5,*)BA11	YP001160
	WRITE(4,333)BA11	YP001170
	IF(BA11.LT.210.AND.BA11.GT.25.OR.BA11.EQ.0)GO TO 240	YP001180
	IF(BA11.EQ.9999) GO TO 730	YP001190
	IF(BA11.EQ.8888)GO TO 710	YP001200
	CALL RANGE(BA11,3,RESP)	YP001210
	IF(RESP.EQ.1)GO TO 210	YP001220
	IF(RESP.EQ.0)GO TO 240	YP001230
	IF(RESP.EQ.9999)GO TO 730	YP001240
	IF(RESP.EQ.8888)GO TO 710	YP001250
240	IF(AGE1.EQ.AGE2)WRITE(10,250)	YP001260
	IF(AGE1.EQ.AGE2)WRITE(4,250)	YP001270
250	FORMAT(/5X,'SPECIFY NUMBER OF TREES PER ACRE IF KNOWN'/	YP001280
:	:13X,'OTHERWISE ENTER 0.'//)	YP001290
	IF(AGE2.GT.AGE1)WRITE(10,260)	YP001300
	IF(AGE2.GT.AGE1)WRITE(4,260)	YP001310
260	FORMAT(/5X,'FOR PROJECTION OF STANDS,'/5X,	YP001320
:	:'ENTER THE KNOWN NUMBER OF TREES OR THE NUMBER OF TREES'/5X,	YP001330
:	:'OBTAINED FROM A PREVIOUSLY PREDICTED STAND TABLE IF POSSIBLE,'	YP001340
:	:/5X,'OTHERWISE ENTER 0.0.'//)	YP001350
	READ(5,*)NT11	YP001360
	WRITE(4,333)NT11	YP001370
	IF(NT11.GT.30.AND.NT11.LT.425)GO TO 280	YP001380
	IF(BA11.EQ.0.AND.NT11.EQ.0)WRITE(10,270)	YP001390
	IF(BA11.EQ.0.AND.NT11.EQ.0)WRITE(4,270)	YP001400
	IF(NT11.EQ.9999) GO TO 730	YP001410
	IF(NT11.EQ.8888)GO TO 710	YP001420

270	FORMAT(/5X,'YOU HAVE INDICATED THAT BOTH BASAL AREA'	YP001430
	:5X,'AND NUMBER OF TREES PER ACRE ARE UNKNOWN.')	YP001440
	IF(BA11.EQ.0.AND.NT11.EQ.0)GO TO 210	YP001450
	IF(NT11.EQ.0)GO TO 280	YP001460
	CALL RANGE(NT11,4,RESP)	YP001470
	IF(RESP.EQ.1)GO TO 240	YP001480
	IF(RESP.EQ.0)GO TO 280	YP001490
	IF(RESP.EQ.9999)GO TO 730	YP001500
	IF(RESP.EQ.8888)GO TO 710	YP001510
280	WRITE(10,290)	YP001520
	WRITE(4,290)	YP001530
290	FORMAT(/5X,'ENTER NUMBER OF PREVIOUS THINNINGS AS:')	YP001540
	:12X,'0 IF STAND HAS NOT BEEN PREVIOUSLY THINNED.')	YP001550
	:12X,'1 IF STAND HAS BEEN PREVIOUSLY THINNED ONCE.')	YP001560
	:12X,'2 IF STAND HAS BEEN PREVIOUSLY THINNED MORE THAN ONCE.')	YP001570
	READ(5,*)TTHINS	YP001580
	WRITE(4,333)TTHINS	YP001590
	IF(TTHINS.EQ.1.OR.TTHINS.EQ.2.OR.TTHINS.EQ.0)GO TO 300	YP001600
	IF(TTHINS.EQ.9999)GO TO 730	YP001610
	IF(TTHINS.EQ.8888)GO TO 710	YP001620
	GO TO 280	YP001630
		YP001640
		YP001650
		YP001660
	-----	YP001670
		YP001680
	*** COMPUTE STAND ATTRIBUTES	YP001690
	-----	YP001700
		YP001710
		YP001720
		YP001730
		YP001740
	COMPUTE INITIAL NUMBER OF TREES PER ACRE	YP001750
		YP001760
300	IF(NT11.EQ.0.)CALL TREES(AGE1,BA11,SITE,TTHINS,NT1)	YP001770
	IF(NT11.GT.0.)NT1=NT11	YP001780
		YP001790
	COMPUTE INITIAL BASAL AREA PER ACRE	YP001800
		YP001810
		YP001820
	IF(BA11.EQ.0.)CALL BASAL(AGE1,SITE,NT1,TTHINS,BA1)	YP001830
	IF(BA11.GT.0.)BA1=BA11	YP001840
		YP001850
		YP001860
		YP001870
		YP001880
310	WRITE(10,320)	YP001890
	WRITE(4,320)	YP001900
320	FORMAT(/5X,'ENTER 1 FOR WHOLE STAND GROWTH AND YIELD ESTIMATES,')	YP001910
	:/8X,'OR 2 FOR DIAMETER DISTRIBUTION LEVEL ESTIMATES.')	YP001920
	READ(5,*)RESP	YP001930
	WRITE(4,333)RESP	YP001940
	IF(RESP.EQ.1)GO TO 330	YP001950
	IF(RESP.EQ.2)GO TO 350	YP001960
	IF(RESP.EQ.9999)GO TO 730	YP001970
	IF(RESP.EQ.8888)GO TO 710	YP001980
	GO TO 310	YP001990
330	IF(TTHINS.EQ.0.OR.TTHINS.EQ.1)BA=EXP((AGE1/AGE2)*ALOG(BA1)	YP002000
	: +4.11893/0.97473*(1.-(AGE1/AGE2))	YP002010
	: +0.01293/0.97473*SITE*(1.-(AGE1/AGE2))	YP002020
	IF(TTHINS.EQ.2)BA=EXP((AGE1/AGE2)*ALOG(BA1)	YP002030
	: +5.84476/0.98858*(1.-(AGE1/AGE2))	YP002040
	: +0.00018/0.98858*SITE*(1.-(AGE1/AGE2))	YP002050
	IF(TTHINS.EQ.0.OR.TTHINS.EQ.1)	YP002060
	: CFV=EXP(5.35740 - 102.45728/SITE - 21.95901/AGE2	YP002070
	: + 0.97473*AGE1/AGE2*ALOG(BA1) + 4.11893*(1.-(AGE1/AGE2)	YP002080
	: + 0.01293*SITE*(1.-(AGE1/AGE2))	YP002090
	IF(TTHINS.EQ.2)	YP002100
	: CFV=EXP(5.33115 - 97.95286/SITE - 25.19324/AGE2	YP002110
	: + 0.98858*AGE1/AGE2*ALOG(BA1) + 5.84476*(1.-(AGE1/AGE2)	YP002120
	: + 0.00018*SITE*(1.-(AGE1/AGE2))	YP002130
	BFV=1363.09165 - 306.96647*BA + 10.26187*CFV	YP002140
	IF(BFV.LT.0.0)BFV=0.0	YP002150

C	WRITE(10,340)AGE1,SITE,AGE2,TTHINS,BA1,CFV,BA,BFV	YP002160
	WRITE(4,340)AGE1,SITE,AGE2,TTHINS,BA1,CFV,BA,BFV	YP002170
	WRITE(11,340)AGE1,SITE,AGE2,TTHINS,BA1,CFV,BA,BFV	YP002180
340	FORMAT(/5X,'WHOLE STAND GROWTH AND YIELD ESTIMATES'/	YP002190
	:5X,41(' '),//,5X,'INITIAL AGE =',F5.0,T40,	YP002200
	: 'SITE INDEX (BASE AGE 50) =',F5.0,/5X,'PROJECTED AGE =',F5.0,T40,	YP002210
	: 'NUMBER OF PREVIOUS THINNINGS =',F3.0,//5X,'INITIAL BASAL AREA =',	YP002220
	:F6.1,T40,'CUBIC-FOOT VOLUME =',F9.1,//5X,	YP002230
	: 'PROJECTED BASAL AREA =',F6.1,T40,'BOARD-FOOT VOLUME =',F9.1////)	YP002240
	GO TO 685	YP002250
C		YP002260
C		YP002270
C		YP002280
C		YP002290
C		YP002300
C	COMPUTE THE FOLLOWING PREDICTED STAND ATTRIBUTES :	YP002310
C	- MINIMUM STAND DIAMETER, DMIN	YP002320
C	- AVERAGE STAND DIAMETER, DAVG	YP002330
C	- AVERAGE SQUARED STAND DIAMETER, D2AVG	YP002340
C	- BASAL AREA PER ACRE, BA	YP002350
C	- AVERAGE HEIGHT OF THE DOMINANTS AND CODOMINANTS, HDOM	YP002360
C		YP002370
C		YP002380
C		YP002390
350	CALL MOMENT(TTHINS,AGE1,AGE2,SITE,BA1,NT1,NT,HDOM,	YP002400
	: DMIN,DAVG,D2AVG,BA)	YP002410
C		YP002420
C		YP002430
C		YP002440
C	COMPUTE PORTION OF HEIGHT EQUATION FOR EASIER CALCULATIONS LATER	YP002450
C	HTCON=3.70051-0.02828*ALOG(BA)-138.35633/AGE2+0.04010*SITE	YP002460
C		YP002470
C		YP002480
C		YP002490
C	-----	YP002500
C	*** CALCULATE ESTIMATES OF THE WEIBULL PARAMETERS	YP002510
C		YP002520
C	-----	YP002530
C	A = LOCATION PARAMETER	YP002540
C	B = SCALE PARAMETER	YP002550
C	C = SHAPE PARAMETER	YP002560
C		YP002570
C	A = DMIN *0.50	YP002580
C	IF(A.LE.0.5) A = 0.5	YP002590
C		YP002600
C		YP002610
C	CALL WEIB(DAVG,D2AVG,A,1.,5.,B,C,X1P,X2P,1ER)	YP002620
C	WRITE(11,123)A,B,C,DAVG,D2AVG	YP002630
123	FORMAT(/5X,'A, B, C, =' ,3F12.8/5X,2F12.4//)	YP002640
C		YP002650
C		YP002660
C		YP002670
C		YP002680
C		YP002690
C		YP002700
C		YP002710
C	-----	YP002720
C	*** GIVEN THE PARAMETER ESTIMATES, DERIVE THE STAND TABLE	YP002730
C		YP002740
C		YP002750
C	-----	YP002760
C		YP002770

C		YP002780
C	DETERMINE THE LARGEST DIAMETER CLASS, DMAX, AS THE LAST	YP002790
C	DIAMETER CLASS CONTAINING AT LEAST 1/2 TREE PER ACRE.	YP002800
C		YP002810
	I=A+0.5	YP002820
	DL=A+0.01	YP002830
	DU=I+0.5	YP002840
360	DDCL=NT*(EXP(-(((DL-A)/B)**C))-EXP(-(((DU-A)/B)**C)))	YP002850
	IF(DFLOAT(I).GT.DAVG.AND.DDCL.LT.0.50)GO TO 370	YP002860
	DMAX=DFLOAT(I)	YP002870
	I=I+1	YP002880
	DU=I+0.5	YP002890
	DL=I-0.5	YP002900
	GO TO 360	YP002910
370	CONTINUE	YP002920
C		YP002930
C		YP002940
C		YP002950
C		YP002960
	WRITE(10,380)	YP002970
	WRITE(4,380)	YP002980
	WRITE(11,380)	YP002990
380	FORMAT(/////)	YP003000
	WRITE(10,390)	YP003010
	WRITE(4,390)	YP003020
	WRITE(11,390)	YP003030
390	FORMAT(22X,'PREDICTED STAND/STOCK TABLE'/)	YP003040
	WRITE(10,400)	YP003050
	WRITE(4,400)	YP003060
	WRITE(11,400)	YP003070
400	FORMAT(T31,'TOTAL',T44,'TOTAL',T54,'INTERNATIONAL 1/4'/	YP003080
	:T2,'DBH',T9,'TREES',T17,'BASAL AREA',T31,'HEIGHT',	YP003090
	:T41,'CUBIC-FOOT',T54,'BOARD-FOOT VOLUME'	YP003100
	:T2,'(IN)',T9,'(AC)',T17,	YP003110
	: '(SQ FT/AC)',T31,'(FEET)',T41,'VOLUME (OB)',	YP003120
	:T54,'11 IN+, 8-IN OB TOP',/72(1-'))	YP003130
C		YP003140
C		YP003150
C	COMPUTE THE PREDICTED DISTRIBUTIONS.	YP003160
C	DCL(I)=TREES IN ITH DIAMETER CLASS	YP003170
C	BCL(I)=BASAL AREA IN ITH DIAMETER CLASS	YP003180
C	VCL(I)=VOLUME IN ITH DIAMETER CLASS	YP003190
C		YP003200
	DSUM = 0.0	YP003210
	BSUM = 0.0	YP003220
	VSUM = 0.0	YP003230
	BVSUM=0.0	YP003240
	DDMIN=0.0	YP003250
	IDMIN=0	YP003260
	I = A + 0.5	YP003270
	DD=DFLOAT(I)	YP003280
	DL = A + 0.01	YP003290
	DU = I + 0.5	YP003300
410	DCL(I) = NT * (EXP(-(((DL-A)/B)**C)) - EXP(-(((DU-A)/B)**C)))	YP003310
	BCL(I) = NT*GAUS(BDIST,DL,DU,10)	YP003320
	VCL(I) = NT*GAUS(VDIST,DL,DU,10)	YP003330
	HTCL=TREHT(DD)	YP003340
	IF(DCL(I).GE.0.10.AND.IDMIN.EQ.0)DDMIN=DFLOAT(I)	YP003350
	IF(DCL(I).GE.0.10)IDMIN=1	YP003360
	IF(DCL(I).LT.0.10)GO TO 440	YP003370
	DSUM = DSUM + DCL(I)	YP003380
	BSUM = BSUM + BCL(I)	YP003390
	VSUM = VSUM + VCL(I)	YP003400
	BVCL(I)=0.0	YP003410
	IF(DU.LE.10.5)GO TO 420	YP003420
	TCVOB=0.010309 + 0.002399*DD*DD*HTCL	YP003430
	CVOB8=TCVOB*(1.0-0.40075*(8.0**2.09311/DD**1.88125))	YP003440
	BURKV=CVOB8*(6.1670 + 8.4641*DD/HTCL - 249.2500/HTCL)	YP003450
	BVCL(I)=BURKV*DCL(I)	YP003460
420	BVSUM=BVSUM+BVCL(I)	YP003470
	ODCL(I)=DCL(I)	YP003480
	OBCL(I)=BCL(I)	YP003490
	IVINT=VCL(I)+0.5	YP003500
	IBVINT=BVCL(I)+0.5	YP003510
	HTCL=HTCL+.5	YP003520



```

WRITE(10,430) I,DCL(I),BCL(I),IHTCL,IVINT,IBVINT
WRITE(4,430) I,DCL(I),BCL(I),IHTCL,IVINT,IBVINT
WRITE(11,430) I,DCL(I),BCL(I),IHTCL,IVINT,IBVINT
430 FORMAT(T3,I2,T8,F6.1,T16,F8.1,T30,I5,T40,I8,T54,I10)
440 I = I + 1
DD=FLOAT(I)
DU = I + 0.5
DL = I - 0.5
IF(DFLOAT(I).LT.DMAX)GO TO 410
DU=DU+5.0
DCL(I)=NT-DSUM
BCL(I)=BA-BSUM
VCL(I)=NT*GAUS(VDIST,DL,DU,10)
DD=FLOAT(I)
HTCL=TREHT(DD)
BVCL(I)=0.
IF(DMAX.LE.10.5)GO TO 445
TCVOB=0.010309 + 0.002399*DD*DD*HTCL
CVOB8=TCVOB*(1.0-0.40075*(8.0**2.09311/DD**1.88125))
BURKV=CVOB8*(6.1670 + 8.4641*DD/HTCL - 249.2500/HTCL)
BVCL(I)=BURKV*DCL(I)
445 ODCL(I)=DCL(I)
OBCL(I)=BCL(I)
IVINT=VCL(I)+0.5
IBVINT=BVCL(I)+0.5
IHTCL=HTCL+.5
WRITE(10,430) I,DCL(I),BCL(I),IHTCL,IVINT,IBVINT
WRITE(4,430) I,DCL(I),BCL(I),IHTCL,IVINT,IBVINT
WRITE(11,430) I,DCL(I),BCL(I),IHTCL,IVINT,IBVINT
DSUM=DSUM+DCL(I)
BSUM=BSUM+BCL(I)
VSUM=VSUM+VCL(I)
BVSUM=BVSUM+BVCL(I)
QAVG=(BSUM/((0.005454154*DSUM))**.5
IVSUM=VSUM+0.5
IBVSUM=BVSUM+0.5
WRITE(10,450) DSUM,BSUM,IVSUM,IBVSUM
WRITE(4,450) DSUM,BSUM,IVSUM,IBVSUM
WRITE(11,450) DSUM,BSUM,IVSUM,IBVSUM
450 FORMAT('2(-)',/'TOTAL',T7,F7.1,T16,F8.1,T32,3(' - '),
:T40,I8,T54,I10)

```

```

YP003530
YP003540
YP003550
YP003560
YP003570
YP003580
YP003590
YP003600
YP003610
YP003620
YP003630
YP003640
YP003650
YP003660
YP003670
YP003680
YP003690
YP003700
YP003710
YP003720
YP003730
YP003740
YP003750
YP003760
YP003770
YP003780
YP003790
YP003800
YP003810
YP003820
YP003830
YP003840
YP003850
YP003860
YP003870
YP003880
YP003890
YP003900
YP003910
YP003920
YP003930
YP003940
YP003950
YP003960
YP003970
YP003980
YP003990
YP004000
YP004010
YP004020
YP004030
YP004040
YP004050
YP004060
YP004070
YP004080
YP004090
YP004100
YP004110
YP004120
YP004130
YP004140
YP004150
YP004160
YP004170
YP004180
YP004190
YP004200
YP004210
YP004220
YP004230
YP004240
YP004250
YP004260

```

C  
C

```

IAGE1=AGE1+.5
IBSUM=BSUM+.5
IAGE2=AGE2+.5
IDSUM=DSUM+.5
ISITE=SITE+.5
IBA11=BA11+.5
INT11=NT11+.5
ITHIN=TTHINS
IHDOM=HDOM+.5
WRITE(10,460) IAGE1,IBSUM,IAGE2,IDSUM,ISITE,DDMIN,IBA11,QAVG,
:INT11,DMAX,ITHIN,IHDOM,IVSUM,IBVSUM
WRITE(11,460) IAGE1,IBSUM,IAGE2,IDSUM,ISITE,DDMIN,IBA11,QAVG,
:INT11,DMAX,ITHIN,IHDOM,IVSUM,IBVSUM
WRITE(4,460) IAGE1,IBSUM,IAGE2,IDSUM,ISITE,DDMIN,IBA11,QAVG,
:INT11,DMAX,ITHIN,IHDOM,IVSUM,IBVSUM
460 FORMAT(////,5X,'STAND TABLE SUMMARY'/5X,19(' - '))
: 'INPUT SUMMARY:',T39,'PROJECTION SUMMARY: '///
: 'INITIAL AGE =',I3,T39,
: 'BASAL AREA (SQ FT/AC) =',I4/
: 'PROJECTED AGE =',I3,T39,
: 'NUMBER OF TREES (/AC)=',I4/
: 'SITE INDEX (FT,BASE AGE 50 FT) =',I4,T39,
: 'MINIMUM DIAMETER (IN) =',F5.1/
: 'INITIAL BASAL AREA (SQ FT/AC) =',I4,T39,
: 'QUADRATIC MEAN DIAMETER (IN) =',F5.1/
: 'NUMBER OF TREES (/AC) =',I4,T39,
: 'MAXIMUM DIAMETER (IN) =',F5.1,/
: 'NUMBER OF',T39,'AVERAGE HEIGHT OF DOMINANTS',/
: 'PREVIOUS THINNINGS =',I4,T44,
: 'AND CODOMINANTS (FT) =',I4,/T39,
: 'CUBIC-FOOT VOLUME =',I8,/T39,

```

	: 'BOARD-FOOT VOLUME =' ,18/T39,	YP004270
	: '(11 IN+ TO AN 8-IN OB TOP)' ///	YP004280
C		YP004290
C		YP004300
C		YP004310
C		YP004320
C	470 WRITE(10,480)	YP004330
	WRITE(4,480)	YP004340
	480 FORMAT(/////5X,'DO YOU WANT TO THIN THE STAND AT THIS TIME?'//	YP004350
	:10X,'ENTER 1 FOR YES' /16X,'0 FOR NO'//)	YP004360
	READ(5,*)DTHIN	YP004370
	WRITE(4,333)DTHIN	YP004380
	IF(DTHIN.EQ.0) GO TO 690	YP004390
	IF(DTHIN.EQ.1)GO TO 490	YP004400
	IF(DTHIN.EQ.9999) GO TO 730	YP004410
	IF(DTHIN.EQ.8888)GO TO 710	YP004420
	GO TO 470	YP004430
		YP004440
		YP004450
		YP004460
	-----	YP004470
	*** THIN THE PREDICTED STAND TABLE	YP004480
		YP004490
	-----	YP004500
		YP004510
		YP004520
		YP004530
	490 KK=DDMIN+0.5	YP004540
	JJ=DMAX+0.5	YP004550
C		YP004560
	500 KKK=0	YP004570
	IJ=0	YP004580
	WRITE(10,510)	YP004590
	WRITE(4,510)	YP004600
	510 FORMAT(/5X,'SPECIFY THE RESIDUAL BASAL AREA DESIRED')	YP004610
	520 CONTINUE	YP004620
	READ(5,*)RESID	YP004630
	WRITE(4,333)RESID	YP004640
	IF(RESID.EQ.9999)GO TO 730	YP004650
	IF(RESID.EQ.8888)GO TO 710	YP004660
	IF(RESID.GT.BSUM)WRITE(10,530)BSUM	YP004670
	IF(RESID.GT.BSUM)WRITE(4,530)BSUM	YP004680
	530 FORMAT(/5X,'RESIDUAL BASAL AREA SPECIFIED IS GREATER THAN'//	YP004690
	: 5X,'CURRENT BASAL AREA OF STAND. REENTER ANOTHER' /5X,	YP004700
	: 'RESIDUAL BASAL AREA VALUE.'//10X,'CURRENT BASAL AREA =' ,F8.2)	YP004710
	IF(RESID.GT.BSUM)GO TO 520	YP004720
	IF(RESID.LE.0)WRITE(10,540)BSUM	YP004730
	IF(RESID.LE.0)WRITE(4,540)BSUM	YP004740
	540 FORMAT(/5X,'RESIDUAL BASAL AREA MUST BE GREATER THAN ZERO.'//5X,	YP004750
	: 'REENTER RESIDUAL BASAL AREA.'//10X,'CURRENT BASAL AREA =' ,F8.2)	YP004760
	IF(RESID.LE.0)GO TO 520	YP004770
		YP004780
		YP004790
	RRBA=BASAL AREA TO REMOVE IN THINNING	YP004800
		YP004810
		YP004820
	RRBA=BSUM-RESID	YP004830
	IIMIN=0	YP004840
	TBREM=0.0	YP004850
	TFBKEP=0.0	YP004860
	TFTKEP=0.0	YP004870
	TFVKEP=0.0	YP004880
	TBVKEP=0.0	YP004890
	WRITE(10,380)	YP004900
	WRITE(4,380)	YP004910
	WRITE(11,380)	YP004920
	WRITE(10,550)	YP004930
	WRITE(4,550)	YP004940
	WRITE(11,550)	YP004950
	550 FORMAT(23X,'STAND/STOCK TABLE AFTER THINNING'//)	

	WRITE(4,400)	YP004960
	WRITE(10,400)	YP004970
	WRITE(11,400)	YP004980
C		YP004990
C	REMOVE NUMBER OF TREES, BASAL AREA, AND VOLUME FROM EACH	YP005000
C	DIAMETER CLASS ACCORDING TO THE THINNING ALGORITHM	YP005010
C		YP005020
	570 DO 580 I=KK,JJ	YP005030
	CALL BACL(D2AVG,I,DCL,BCL,PROP,RNT,RBA,BAR,RNTR,RRBA,TBREM,KKK,	YP005040
	:IJ,OBCL,TTTHINS)	YP005050
C		YP005060
C	BAREM ACCUMULATES BA REMOVED/CLASS	YP005070
C	TBREM ACCUMULATES TOTAL BA REMOVED OVER ALL CLASS	YP005080
C	TREM ACCUMULATES NT REMOVED/CLASS	YP005090
C		YP005100
	BAREM(I)=BAREM(I)+BAR(I)	YP005110
	TBREM=TBREM+BAR(I)	YP005120
	TREM(I)=TREM(I)+RNTR(I)	YP005130
	IF(TBREM.GE.RRBA)IJ=1	YP005140
	580 CONTINUE	YP005150
C		YP005160
	IF(TBREM.LT.RRBA)KKK=1	YP005170
	IF(TBREM.LT.RRBA)GO TO 570	YP005180
C		YP005190
C	FBKEP ACCUMULATES BASAL AREA KEPT/CLASS	YP005200
C	FTKEP ACCUMULATES NUMBER OF TREES KEPT/CLASS	YP005210
C	FVKEP ACCUMULATES CUBIC-FOOT VOLUME KEPT/CLASS	YP005220
C	FVREM ACCUMULATES CUBIC-FOOT VOLUME REMOVED/CLASS	YP005230
C	FBVKEP ACCUMULATES BOARD-FOOT VOLUME KEPT/CLASS	YP005240
C	FBVREM ACCUMULATES BOARD-FOOT VOLUME REMOVED/CLASS	YP005250
C	TFBKEP AND TFTKEP ACCUMULATE BA AND NT KEPT OVER ALL CLASSES	YP005260
C	TFVKEP AND TBVKEP ACCUMULATE VOLUMES KEPT OVER ALL CLASSES	YP005270
C		YP005280
	DO 620 I=KK,JJ	YP005290
	DD=FLOAT(I)	YP005300
	IHTCL=TREEHT(DD)+ 0.5	YP005310
	FBKEP=OBCL(I)-BAREM(I)	YP005320
	FTKEP=ODCL(I)-TREM(I)	YP005330
	IF(ODCL(I).GT.0)FVKEP=(FTKEP/ODCL(I))*VCL(I)	YP005340
	IF(ODCL(I).EQ.0)FVKEP=0.0	YP005350
	IF(ODCL(I).GT.0)FBVKEP=(FTKEP/ODCL(I))*BVCL(I)	YP005360
	IF(ODCL(I).EQ.0)FBVKEP=0.0	YP005370
	IF(FTKEP.GE.0.1.AND.FBKEP.GE.0.01)GO TO 600	YP005380
	IF(FBKEP.GE.0.01)GO TO 590	YP005390
	FTKEP=0.0	YP005400
	FBKEP=0.0	YP005410
	FVKEP=0.0	YP005420
	FBVKEP=0.0	YP005430
	GO TO 600	YP005440
590	FTKEP=FBKEP/(0.005454154*DFLOAT(I))*DFLOAT(I)	YP005450
	IF(ODCL(I).GT.0.0)FVKEP=(FTKEP/ODCL(I))*VCL(I)	YP005460
	IF(ODCL(I).EQ.0)FVKEP=0.0	YP005470
	IF(ODCL(I).GT.0.0)FBVKEP=(FTKEP/ODCL(I))*BVCL(I)	YP005480
	IF(ODCL(I).EQ.0)FBVKEP=0.0	YP005490
600	TFBKEP=TFBKEP+FBKEP	YP005500
	TFTKEP=TFTKEP+FTKEP	YP005510
	TFVKEP=TFVKEP+FVKEP	YP005520
	TBVKEP=TBVKEP+FBVKEP	YP005530
	IF(FTKEP.GE.0.1.AND.IIMIN.EQ.0)DDMIN=I	YP005540
	IF(FTKEP.GE.0.1)IIMIN=1	YP005550
	IVKEP=FVKEP+0.5	YP005560
	IBVKEP=FBVKEP+0.5	YP005570
	IF(FTKEP.EQ.0.0)IHTCL=0	YP005580
	WRITE(10,430)I,FTKEP,FBKEP,IHTCL,IVKEP,IBVKEP	YP005590
	WRITE(4,430)I,FTKEP,FBKEP,IHTCL,IVKEP,IBVKEP	YP005600
	WRITE(11,430)I,FTKEP,FBKEP,IHTCL,IVKEP,IBVKEP	YP005610
620	CONTINUE	YP005620
	QQAVG=(TFBKEP/(TFTKEP*0.005454154))*0.5	YP005630
	ITVKEP=TFVKEP+0.5	YP005640
	ITBKEP=TBVKEP+0.5	YP005650
	WRITE(10,450)TFTKEP,TFBKEP,ITVKEP,ITBKEP	YP005660
	WRITE(4,450)TFTKEP,TFBKEP,ITVKEP,ITBKEP	YP005670
	WRITE(11,450)TFTKEP,TFBKEP,ITVKEP,ITBKEP	YP005680
		YP005690

C			YP005700
C			YP005710
			YP005720
			YP005730
			YP005740
			YP005750
			YP005760
			YP005770
			YP005780
			YP005790
			YP005800
			YP005810
			YP005820
			YP005830
			YP005840
			YP005850
			YP005860
			YP005870
			YP005880
			YP005890
			YP005900
			YP005910
			YP005920
			YP005930
			YP005940
			YP005950
			YP005960
			YP005970
			YP005980
			YP005990
			YP006000
			YP006010
			YP006020
			YP006030
			YP006040
			YP006050
			YP006060
			YP006070
			YP006080
			YP006090
			YP006100
			YP006110
			YP006120
			YP006130
			YP006140
			YP006150
			YP006160
			YP006170
			YP006180
			YP006190
			YP006200
			YP006210
			YP006220
			YP006230
			YP006240
			YP006250
			YP006260
			YP006270
			YP006280
			YP006290
			YP006300
			YP006310
			YP006320
			YP006330
			YP006340
			YP006350
			YP006360
			YP006370
			YP006380
			YP006390
			YP006400

```

C
C
TTREM=DSUM-TFTKEP
IVREM=VSUM-TFVKEP+0.5
IBVREM=BVSUM-TBVKEP+0.5
IVSUM=VSUM+0.5
IBVSUM=IBVSUM+0.5

C
C
C
IAGE2=AGE2+.5
ISITE=SITE+.5
ITFBK=TFBKEP+.5
IDSUM=DSUM+.5
ITTREM=TTREM+.5
IBSUM=BSUM+.5
IRRBA=RRBA+.5
WRITE(10,640) IAGE2, ITFBK, ISITE, ITVKEP, DDMIN, ITBKEP, QQAVG, DMAX,
: IDSUM, ITTREM, IBSUM, IRRBA, IVSUM, IVREM, IBVSUM, IBVREM
WRITE(11,640) IAGE2, ITFBK, ISITE, ITVKEP, DDMIN, ITBKEP, QQAVG, DMAX,
: IDSUM, ITTREM, IBSUM, IRRBA, IVSUM, IVREM, IBVSUM, IBVREM
WRITE(4,640) IAGE2, ITFBK, ISITE, ITVKEP, DDMIN, ITBKEP, QQAVG, DMAX,
: IDSUM, ITTREM, IBSUM, IRRBA, IVSUM, IVREM, IBVSUM, IBVREM
640 FORMAT(///,3X,'STAND TABLE SUMMARY AFTER THINNING'/3X,34('-')//
: 3X,'AGE =' ,13,
: T44,'BASAL AREA (SQ FT/AC) =' ,15/,
: 3X,'SITE INDEX (FT, BASE AGE 50 FT) =' ,14,
: T44,'CUBIC-FOOT VOLUME =' ,18/,
: 3X,'MINIMUM DIAMETER (IN) =' ,F5.1,
: T44,'BOARD-FOOT VOLUME =' ,18/,
: 3X,'QUADRATIC MEAN DIAMETER (IN) =' ,F5.1,
: T44,'(11 IN+ TO AN 8-IN OB TOP) ',
: 3X,'MAXIMUM DIAMETER (IN) =' ,F5.1,
: ///3X,'NUMBER OF TREES (/AC) PRIOR TO THINNING =' ,15/,
: 3X,'NUMBER OF TREES (/AC) REMOVED IN THINNING =' ,15,
: ///3X,'BASAL AREA (SQ FT/AC) PRIOR TO THINNING =' ,15,
: 3X,'BASAL AREA (SQ FT/AC) REMOVED IN THINNING =' ,15,
: ///3X,'CUBIC-FOOT VOLUME PRIOR TO THINNING =' ,18,
: 3X,'CUBIC-FOOT VOLUME REMOVED IN THINNING =' ,18,
: ///3X,'BOARD-FOOT VOLUME PRIOR TO THINNING =' ,18,
: 3X,'BOARD-FOOT VOLUME REMOVED IN THINNING =' ,18/////))

C
C
C
WRITE(11,380)

650 WRITE(10,660)
WRITE(4,660)
660 FORMAT(//5X,'DO YOU WANT TO RETHIN THE PREDICTED STAND TABLE',
: /5X,'TO ANOTHER LEVEL OF RESIDUAL BASAL AREA ?',//
: 5X,'ENTER 1 FOR YES'/11X,'0 FOR NO')
READ(5,*)RESP
WRITE(4,333)RESP
IF(RESP.EQ.0)GO TO 690
IF(RESP.EQ.9999)GO TO 730
IF(RESP.EQ.8888)GO TO 710
IF(RESP.NE.1)GO TO 650

C
C
C
REINITIALIZE VARIABLES TO RETHIN STAND TABLE

KKK=0
IJ=0
DO 670 I=KK,JJ
BAR(I)=0.0
RNTR(I)=0.0
BAREM(I)=0.0
TREM(I)=0.0
PROP(I)=0.0
DCL(I)=ODCL(I)
BCL(I)=OBCL(I)
670 CONTINUE

```

	WRITE(10,680)RESID	YP006410
	WRITE(4,680)RESID	YP006420
680	FORMAT(/5X,'THE RESIDUAL BASAL AREA PREVIOUSLY SPECIFIED WAS',	YP006430
	:F7.2/)	YP006440
	GO TO 500	YP006450
-----		YP006460
C		YP006470
C		YP006480
C		YP006490
685	WRITE(4,686)	YP006500
	WRITE(10,686)	YP006510
	WRITE(11,686)	YP006520
686	FORMAT(/5X,'DO YOU WANT THE CORRESPONDING STAND TABLE ?'	YP006530
	://10X,'ENTER 1 FOR YES'/16X,'0 FOR NO')	YP006540
	READ(5,*)RESP	YP006550
	WRITE(4,333)RESP	YP006560
	IF(RESP.EQ.1)GO TO 350	YP006570
	IF(RESP.EQ.0.)GO TO 690	YP006580
	IF(RESP.EQ.8888)GO TO 710	YP006590
	IF(RESP.EQ.9999)GO TO 730	YP006600
	GO TO 685	YP006610
690	WRITE(10,700)	YP006620
	WRITE(4,700)	YP006630
700	FORMAT(/5X,'DO YOU WANT TO MAKE ANOTHER PROJECTION ?'	YP006640
	://10X,'ENTER 1 FOR YES'/16X,'0 FOR NO')	YP006650
	READ(5,*)RESP	YP006660
	WRITE(4,333)RESP	YP006670
	IF(RESP.EQ.1)GO TO 110	YP006680
	IF(RESP.EQ.8888)GO TO 710	YP006690
	IF(RESP.EQ.0.OR.RESP.EQ.9999)GO TO 730	YP006700
	GO TO 690	YP006710
710	WRITE(10,720)	YP006720
	WRITE(4,720)	YP006730
720	FORMAT(/5X,'PROGRAM HAS BEEN RESTARTED'///)	YP006740
	GO TO 110	YP006750
730	WRITE(10,740)	YP006760
	WRITE(4,740)	YP006770
740	FORMAT(/5X,'THE PROGRAM HAS BEEN TERMINATED BY THE USER'///)	YP006780
	STOP	YP006790
	END	YP006800
C		YP006810
C		YP006820
C		YP006830
C		YP006840
C		YP006850
C	-----	YP006860
C	*** SUBROUTINES AND FUNCTIONS	YP006870
C		YP006880
C		YP006890
C	-----	YP006900
C		YP006910
C		YP006920
C		YP006930
C		YP006940
C	SUBROUTINE RANGE(SICK,IVAR,RESP)	YP006950
C		YP006960
	IF((IVAR.EQ.1.AND.(SICK.LT.5.OR.SICK.GT.150)).OR.	YP006970
	: (IVAR.EQ.2.AND.(SICK.LT.40.OR.SICK.GT.200)).OR.	YP006980
	: (IVAR.EQ.3.AND.(SICK.LT.15.OR.SICK.GT.300)).OR.	YP006990
	: (IVAR.EQ.4.AND.(SICK.LT.15.OR.SICK.GT.600)))GO TO 170	YP007000
	WRITE(10,100)SICK	YP007010
	WRITE(4,100)SICK	YP007020
100	FORMAT(/5X,'WARNING: SPECIFIED VALUE OF',F7.1,2X,	YP007030
	: 'IS BEYOND DATA RANGE,/'	YP007040
	:14X,'ILLOGICAL OR INCONSISTENT RESULTS MAY BE OBTAINED.')	YP007050
	IF(IVAR.EQ.1)WRITE(4,110)	YP007060
	IF(IVAR.EQ.2)WRITE(4,120)	YP007070
	IF(IVAR.EQ.3)WRITE(4,130)	YP007080
	IF(IVAR.EQ.4)WRITE(4,140)	YP007090
	IF(IVAR.EQ.1)WRITE(10,110)	YP007100
	IF(IVAR.EQ.2)WRITE(10,120)	YP007110
	IF(IVAR.EQ.3)WRITE(10,130)	YP007120
	IF(IVAR.EQ.4)WRITE(10,140)	YP007130

110	FORMAT(/14X,'DATA RANGE FOR AGE IS FROM 15 TO 90 YEARS,')	YP007140
120	FORMAT(/14X,'DATA RANGE FOR SITE INDEX IS FROM 75 TO 140 FEET,')	YP007150
130	FORMAT(/14X,'DATA RANGE FOR BASAL AREA IS FROM 25 TO 210 SQ.FT.,')	YP007160
140	FORMAT(/14X,'DATA RANGE FOR TREES PER ACRE IS 30 TO 425,')	YP007170
150	WRITE(10,160)	YP007180
160	FORMAT(14X,'DO YOU WANT TO SPECIFY ANOTHER VALUE?'/	YP007190
	:23X,'ENTER 1 FOR YES'/	YP007200
	:30X,'0 FOR NO')	YP007210
	READ(5,*)RESP	YP007220
	WRITE(4,333)RESP	YP007230
333	FORMAT(/5X,F5.0/)	YP007240
	IF(RESP.NE.0.AND.RESP.NE.1.AND.RESP.NE.8888.AND.	YP007250
	:RESP.NE.9999)GO TO 150	YP007260
	RETURN	YP007270
170	WRITE(10,180)SICK	YP007280
	IF(IVAR.EQ.1)WRITE(4,110)	YP007290
	IF(IVAR.EQ.2)WRITE(4,120)	YP007300
	IF(IVAR.EQ.3)WRITE(4,130)	YP007310
	IF(IVAR.EQ.4)WRITE(4,140)	YP007320
	IF(IVAR.EQ.1)WRITE(10,110)	YP007330
	IF(IVAR.EQ.2)WRITE(10,120)	YP007340
	IF(IVAR.EQ.3)WRITE(10,130)	YP007350
	IF(IVAR.EQ.4)WRITE(10,140)	YP007360
	WRITE(10,190)	YP007370
	WRITE(4,190)	YP007380
180	FORMAT(/14X,'SPECIFIED VALUE OF',F7.1,2X,'IS EXTREME')	YP007390
190	FORMAT(14X,'YOU MUST SPECIFY ANOTHER VALUE TO CONTINUE'/)	YP007400
	RESP=1	YP007410
	RETURN	YP007420
	END	YP007430
		YP007440
		YP007450
	CALCULATE TREES PER ACRE GIVEN AGE, SITE, BASAL AREA,	YP007460
	AND NUMBER OF PREVIOUS THINNINGS	YP007470
		YP007480
	SUBROUTINE TREES(AGE1,BA11,SITE,TTHINS,NT1)	YP007490
	REAL*4 NT1	YP007500
	IF(TTHINS.EQ.0)	YP007510
	: TNT=6.43346+38.24834/AGE1-0.01309*SITE-67.25874/BA11	YP007520
	IF(TTHINS.EQ.1)	YP007530
	: TNT=6.12444+59.93859/AGE1-0.01911*SITE-73.59987/BA11	YP007540
	IF(TTHINS.EQ.2)	YP007550
	: TNT=6.12335+69.03772/AGE1-0.02083*SITE-78.12201/BA11	YP007560
	NT1=EXP(TNT)	YP007570
	RETURN	YP007580
	END	YP007590
		YP007600
		YP007610
		YP007620
	COMPUTE INITIAL BASAL AREA FROM AGE, SITE, NUMBER OF TREES,	YP007630
	AND NUMBER OF PREVIOUS THINNINGS.	YP007640
		YP007650
		YP007660
	SUBROUTINE BASAL(AGE1,SITE,NT1,TTHINS,BA1)	YP007670
	REAL*4 NT1	YP007680
	IF(TTHINS.EQ.0)	YP007690
	: BAS=4.55808-31.21173/AGE1+0.01324*SITE-77.35908/NT1	YP007700
	IF(TTHINS.EQ.1)	YP007710
	: BAS=4.16240-38.13602/AGE1+0.01606*SITE-47.19922/NT1	YP007720
	IF(TTHINS.EQ.2)	YP007730
	: BAS=4.24861-45.83883/AGE1+0.01566*SITE-37.78880/NT1	YP007740
	BA1=EXP(BAS)	YP007750
	RETURN	YP007760
	END	YP007770
		YP007780
		YP007790
		YP007800
		YP007810

C	COMPUTE MINIMUM, AVERAGE, AND AVERAGE SQUARED DIAMETER,	YP007820
C	AVERAGE HEIGHT OF THE DOMINANTS AND CODOMINANTS, AND	YP007830
C	BASAL AREA FROM THE SPECIFIED INPUT VARIABLES.	YP007840
C		YP007850
C	SUBROUTINE MOMENT(TTHINS,AGE1,AGE2,SITE,BA1,NT1,NT,HDOM,	YP007860
	: DMIN,DAVG,D2AVG,BA)	YP007870
	REAL*4 NT1,NT	YP007880
	IF (AGE1.LT.AGE2)GO TO 100	YP007890
	BA=BA1	YP007900
	NT=NT1	YP007910
	GO TO 110	YP007920
100	IF(TTHINS.EQ.0.OR.TTHINS.EQ.1)BA3=(AGE1/AGE2)*ALOG(BA1)	YP007930
	: +4.11893/0.97473*(1.-(AGE1/AGE2))	YP007940
	: +0.01293/0.97473*SITE*(1.-(AGE1/AGE2))	YP007950
	IF(TTHINS.EQ.2)BA3=(AGE1/AGE2)*ALOG(BA1)	YP007960
	: +5.84476/0.98858*(1.-(AGE1/AGE2))	YP007970
	: +0.00018/0.98858*SITE*(1.-(AGE1/AGE2))	YP007980
	BA=EXP(BA3)	YP007990
	IF(TTHINS.GT.0)NT=NT1	YP008000
	IF(TTHINS.EQ.0)CALL TREES(AGE2,BA,SITE,TTHINS,NT)	YP008010
110	D2AVG=BA/(0.005454154*NT)	YP008020
C		YP008030
C		YP008040
C		YP008050
C	COMPUTE AVERAGE HEIGHT OF THE DOMINANTS AND CODOMINANTS	YP008060
C	FROM THE SITE INDEX EQUATION OF BECK(1975)	YP008070
C		YP008080
C		YP008090
C	HDOM=EXP(ALOG(SITE) + 21.08707*((1./50)-(1./AGE2)))	YP008100
C		YP008110
C	COMPUTE AVERAGE DIAMETER, DAVG, BY PREDICTING	YP008120
C	LN(VARIANCE OF DIAMETER) AND SOLVING FOR DAVG	YP008130
C		YP008140
C		YP008150
C	IF(TTHINS.EQ.0)	YP008160
	: ALDVAR=-13.408240 + 0.452133*ALOG(BA)+3.059782*ALOG(HDOM)	YP008170
	: - 0.206638*AGE2*NT/10000	YP008180
	IF(TTHINS.EQ.1.OR.TTHINS.EQ.2)	YP008190
	: ALDVAR=-5.201644 + 0.807731*ALOG(BA) + 0.723825*ALOG(HDOM)	YP008200
	: - 0.335597*AGE2*NT/10000	YP008210
	DAVG=(D2AVG-EXP(ALDVAR))*0.5	YP008220
C		YP008230
C		YP008240
C	COMPUTE THE MINIMUM DIAMETER CLASS	YP008250
C		YP008260
C		YP008270
C		YP008280
	IF(TTHINS.EQ.1.OR.TTHINS.EQ.2)GO TO 120	YP008290
	DMIN=5.0	YP008300
	GO TO 130	YP008310
120	DMIN=EXP(1.194388 + 0.056374*((BA/(NT*0.005454154))*0.5)	YP008320
	: + 3.040222/(NT**0.5) - 394.072189/(AGE2*HDOM))	YP008330
130	IF(DMIN.LE.5.0)DMIN=5.0	YP008340
	RETURN	YP008350
	END	YP008360
C		YP008370
C		YP008380
C		YP008390
C	COMPUTE TOTAL TREE HEIGHTS	YP008400
C		YP008410
C		YP008420
C	FUNCTION TREEHT(DBH)	YP008430
	COMMON/AREA2/HTCON, DMAX, HDOM	YP008440
	TREEHT=HDOM/EXP(-0.09675 + ((1./DBH)-(1./DMAX))*HTCON)	YP008450
	RETURN	YP008460
	END	YP008470
C		YP008480
C		YP008490

C			YP008500
C		REMOVE BASAL AREA AND TREES PER ACRE FROM EACH DIAMETER CLASS	YP008510
C			YP008520
		SUBROUTINE BACL(D2AVG, I, DCL, BCL, PROP, RNT, RBA, BAR, RNTR, RRBA, TBREM,	YP008530
		: KKK, IJ, OBCL, TTHINS)	YP008540
		DIMENSION RBA(50), RNT(50), DCL(50), BCL(50), PROP(50), BAR(50),	YP008550
		: RNTR(50), OBCL(50)	YP008560
		IF(IJ.EQ.0)GO TO 100	YP008570
		RNTR(I)=0.0	YP008580
		BAR(I)=0.0	YP008590
		RBA(I)=BCL(I)	YP008600
		RNT(I)=DCL(I)	YP008610
		GO TO 110	YP008620
100		D2=DFLOAT(I)*DFLOAT(I)	YP008630
		IF(TTHINS.EQ.0)TPROP=-0.70406915*((D2/D2AVG)**1.87666308)	YP008640
		IF(TTHINS.EQ.1.OR.TTHINS.EQ.2)	YP008650
		: TPROP=-2.61225530*((D2/D2AVG)**2.00626750)	YP008660
		PRO=EXP(TPROP)	YP008670
		IF(PRO.LT..01)PRO=0.0	YP008680
		IF(KKK.EQ.1)BAR1=BCL(I)	YP008690
		IF(KKK.EQ.0)BAR1=BCL(I)*PRO	YP008700
		BLEFT=RRBA-TBREM	YP008710
		IF(BAR1.LE.BLEFT)BAR(I)=BAR1	YP008720
		IF(BAR1.GT.BLEFT)BAR(I)=BLEFT	YP008730
		RNTR(I)=BAR(I)/(0.005454154*D2)	YP008740
		IF(RNTR(I).GE.DCL(I))RNTR(I)=DCL(I)	YP008750
		RBA(I)=BCL(I)-BAR(I)	YP008760
		IF(RBA(I).LT.0.01)RBA(I)=0.0	YP008770
		IF(OBCL(I).GT.0)PROP(I)=1. - (RBA(I)/OBCL(I))	YP008780
		IF(OBCL(I).EQ.0)PROP(I)=1.	YP008790
		IF(PROP(I).GT.0.99999)BAR(I)=BCL(I)	YP008800
		RNT(I)=DCL(I)-RNTR(I)	YP008810
		IF(RBA(I).LT.0.01)RNT(I)=0.0	YP008820
		IF(RBA(I).LT.0.01)RNTR(I)=DCL(I)	YP008830
		DCL(I)=RNT(I)	YP008840
		BCL(I)=RBA(I)	YP008850
110		RETURN	YP008860
		END	YP008870
C			YP008880
C			YP008890
C			YP008900
		SUBROUTINE WEIB(X1,X2,LOCA,BL,TL,B,C,X1P,X2P,IER)	YP008910
		IMPLICIT REAL*8 (Z)	YP008920
		REAL LOCA	YP008930
		COMMON/AREA3/ZA,ZB,ZC,ZD1,ZD2	YP008940
			YP008950
		CALCULATE B AND C PARAMETERS OF THE WEIBULL DISTRIBUTION	YP008960
		ACCORDING TO METHOD PRESENTED BY BURK AND BURKHART (1984).	YP008970
			YP008980
			YP008990
		PURPOSE	YP009000
		TO RECOVER THE SHAPE AND SCALE PARAMETERS OF THE WEIBULL	YP009010
		USING THE FIRST AND SECOND NONCENTRAL MOMENTS OF DBH.	YP009020
			YP009030
		REMARKS	YP009040
		IER=0	YP009050
		SUCCESSFUL SOLUTION OBTAINED WITH NO CHANGES.	YP009060
		IER=1	YP009070
		ITERATION DID NOT CONVERGE. X2P IS THE VALUE OF X2	YP009080
		CORRESPONDING TO THE SOLUTION OBTAINED. THE USER MUST	YP009090
		DETERMINE IF THIS IS CLOSE ENOUGH TO X2 FOR HIS PURPOSES.	YP009100
		IER=2	YP009110
		SOLUTION OBTAINED AFTER PERTUBATING X1. X1 IS PERTUBATED	YP009120
		IN INCREMENTS OF .01 UNTIL A SOLUTION IN THE ALLOWABLE	YP009130
		RANGE IS FOUND. X1P CONTAINS THE PERTUBATED VALUE OF X1.	YP009140
		IER=3	YP009150
		A SOLUTION IN THE ALLOWABLE RANGE COULD NOT BE FOUND	YP009160
		EVEN UPON PERTUBATING X1.	YP009170
			YP009180
		METHOD	YP009190
		THE SECANT METHOD IS USED FOR ITERATION ON THE SHAPE	YP009200
		PARAMETER.	YP009210



```

C      IER=0
      ZA=DBLE(LOCA)
      B=0.0
      C=0.0
      ZD2=DBLE(X2)
      X1P=X1
      X2P=X2
      IFLAG=0
C
C      ADJUSTMENT OF X1 IS MADE IF THE LOWER AND UPPER SHAPE VALUES
C      DO NOT BRACKET THE SOLUTION: THAT IS, ZFCV IS A STRICTLY
C      INCREASING FUNCTION OF THE SHAPE PARAMETER.
C
10     ZD1=DBLE(X1P)
      ZXN=DBLE(BL)
      ZFXN=ZFCV(ZXN)
      IF(ZFXN.LT.0.D0)GO TO 30
      IER=2
      IF(IFLAG.EQ.0)GO TO 20
      IER=3
      RETURN
20     X1P=X1P+.01
      GO TO 10
30     ZXN1=DBLE(TL)
      ZFXN1=ZFCV(ZXN1)
      IF(ZFXN1.GT.0.D0)GO TO 40
      IER=2
      IFLAG=1
      X1P=X1P-.01
      GO TO 10
C
C      DO 5 BISECTION ITERATIONS TO GET STARTED
C
40     DO 60 J=1,5
      ZTEMP=(ZXN+ZXN1)/2.D0
      ZFTEMP=ZFCV(ZTEMP)
      IF(ZFTEMP*ZFXN.LE.0.D0)GO TO 50
      ZXN=ZTEMP
      ZFXN=ZFTEMP
      GO TO 60
50     ZXN1=ZTEMP
      ZFXN1=ZFTEMP
60     CONTINUE
C
C      BEGIN SECANT ITERATION
C
      DO 70 J=1,100
      ZTEMP=ZXN-ZFXN*(ZXN-ZXN1)/(ZFXN-ZFXN1)
      ZXN1=ZXN
      ZFXN1=ZFXN
      ZXN=ZTEMP
      ZFXN=ZFCV(ZXN)
      IF(DABS(ZFXN).LE.0.00001D0)GO TO 80
70     CONTINUE
      IER=1
      X2P=XD2-ZFXN
80     C=ZC
      B=ZB
C      WRITE(10,*)X1,X2,B,C,IER
      RETURN
      END
C
C      DOUBLE PRECISION FUNCTION ZFCV(ZX)
      IMPLICIT REAL*8 (Z)
      COMMON/AREA3/ZA,ZB,ZC,ZD1,ZD2
C
C      THIS FUNCTION EVALUATES THE FUNCTION WHOSE ROOT IS DESIRED.
C
      ZC=ZX
      ZG1=DGAMMA(1.D0+1.D0/ZC)
      ZG2=DGAMMA(1.D0+2.D0/ZC)

```

```

YP009220
YP009230
YP009240
YP009250
YP009260
YP009270
YP009280
YP009290
YP009300
YP009310
YP009320
YP009330
YP009340
YP009350
YP009360
YP009370
YP009380
YP009390
YP009400
YP009410
YP009420
YP009430
YP009440
YP009450
YP009460
YP009470
YP009480
YP009490
YP009500
YP009510
YP009520
YP009530
YP009540
YP009550
YP009560
YP009570
YP009580
YP009590
YP009600
YP009610
YP009620
YP009630
YP009640
YP009650
YP009660
YP009670
YP009680
YP009690
YP009700
YP009710
YP009720
YP009730
YP009740
YP009750
YP009760
YP009770
YP009780
YP009790
YP009800
YP009810
YP009820
YP009830
YP009840
YP009850
YP009860
YP009870
YP009880
YP009890
YP009900
YP009910
YP009920
YP009930
YP009940

```

	ZB=(ZD1-ZA)/ZG1	YP009950
	ZFCV=ZD2-ZA*ZA-2.D0*ZA*ZB*ZG1-ZB*ZB*ZG2	YP009960
	RETURN	YP009970
	END	YP009980
C		YP009990
C		YP010000
C		YP010010
C		YP010020
C		YP010030
	FUNCTION BDIST(DBH)	YP010040
	REAL*4 NT	YP010050
	COMMON/AREA1/DAVG,D2AVG,A,B,C	YP010060
	BDIST = 0.0	YP010070
	XX = 1.0	YP010080
	XY = C * ALOG((DBH-A)/B)	YP010090
	IF(XY.GT.4.0) RETURN	YP010100
	IF(XY.LT.-10.0) GO TO 100	YP010110
	XX = EXP(-(((DBH-A)/B)**C))	YP010120
100	BDIST = 0.005454154*DBH*DBH*C/B*((DBH-A)/B)**(C-1.0)*XX	YP010130
	RETURN	YP010140
	END	YP010150
C		YP010160
C		YP010170
C		YP010180
C	NUMERICAL INTEGRATION OF THE FUNCTION VDIST GIVES	YP010190
C	CUBIC-FOOT VOLUME IN A SPECIFIED DIAMETER CLASS	YP010200
C		YP010210
C		YP010220
	FUNCTION VDIST(DBH)	YP010230
	REAL*4 NT	YP010240
	COMMON/AREA1/DAVG,D2AVG,A,B,C	YP010250
	EXTERNAL TREEHT	YP010260
	VDIST = 0.0	YP010270
	XX = 1.0	YP010280
	XY = C * ALOG((DBH-A)/B)	YP010290
	IF(XY.GT.4.0) RETURN	YP010300
	IF(XY.LT.-10.0) GO TO 100	YP010310
	XX = EXP(-(((DBH-A)/B)**C))	YP010320
100	VDIST = (0.010309+0.002399*DBH*DBH*TREEHT(DBH))*C/B	YP010330
	:(DBH-A)/B)**(C-1.0)*XX	YP010340
	RETURN	YP010350
	END	YP010360
C		YP010370
C		YP010380
C		YP010390
C	FUNCTION GAUS (F,A,B,N)	YP010400
	DIMENSION C(10),D(10)	YP010410
	EXTERNAL F	YP010420
C		YP010430
	DATA C/.076526521,.22778585,.37370609,.510867,	YP010440
	: .63605368,.74633191,.83911697,.91223443,	YP010450
	: .96397193,.9931286/	YP010460
C		YP010470
	DATA D/.15275339,.14917299,.14209611,.13168864,	YP010480
	: .11819453,.10193012,.083276742,.062672048,	YP010490
	: .04060143,.017614007/	YP010500
C		YP010510
	S=(B-A)/N/2	YP010520
	T=A+S	YP010530
	GI=0	YP010540
C		YP010550
C	COMPUTE INTEGRAL FOR EACH SUBINTERVAL	YP010560
C		YP010570
	DO 100 J=1,N	YP010580
	P=0	YP010590
C		YP010600
C	COMPUTE SUMMATION FACTOR FOR EACH SUBINTERVAL	YP010610
C		YP010620
C		YP010630
	DO 200 K=1,10	YP010640
	P=P+D(K)*(F(S*C(K)+T)+F(T-S*C(K)))	YP010650
200	CONTINUE	YP010660
	GI=GI+P*S	YP010670
	T=T+2*S	YP010680
100	CONTINUE	YP010690
	GAUS=GI	YP010700
C		YP010710
	RETURN	YP010720
	END	YP010730

Copies of FOREST SCIENCE MONOGRAPHS as available may be obtained at \$5.00 per copy, postpaid (except Monograph 24 at \$6.00), from Society of American Foresters, 5400 Grosvenor Lane, Bethesda, MD 20814.

- No. 1. Private Forestry in Norway—A Case Study in Small Woodland Management and Policy. By John A. Zivnuska. 1959. 49 pages.
- No. 2. Racial Variation in Ponderosa Pine. By A. E. Squillace and Roy R. Silen. 1962. 27 pages.
- No. 3. Continuous Forest Inventory With Partial Replacement of Samples. By Kenneth D. Ware and Tiberius Cunia. 1962. 40 pages. (Out of print.)
- No. 4. Optical Dendrometers For Out-of-Reach Diameters: A Conspectus And Some New Theory. By L. R. Grosenbaugh. 1963. 47 pages. (Out of print.)
- No. 5. Stem Form Development of Forest Trees. By Philip R. Larson. 1963. 42 pages. (Out of print.)
- No. 6. Lammas Growth and Prolepsis in Jack Pine in the Lakes States. By Thomas D. Rudolph. 1964. 70 pages.
- No. 7. The Carrying Capacity of Wild Lands for Recreation. By J. Alan Wagar. 1964. 24 pages. (Out of print.)
- No. 8. Some Forest Types of Central Newfoundland and Their Relation to Environmental Factors. By A. W. H. Damman. 1964. 62 pages.
- No. 9. Dry-Matter Production in Immature Balsam Fir Stands. By G. L. Baskerville. 1965. 42 pages. (Out of print.)
- No. 10. Geographic Variation in Slash Pine. By A. E. Squillace. 1966. 56 pages.
- No. 11. Geographic Variation in Survival, Growth, and Fusiform Rust Infection of Planted Loblolly Pine. By Osborn O. Wells and Philip C. Wakeley. 1966. 40 pages.
- No. 12. A Dynamic Programming-Markov Chain Approach to Forest Production Control. By James N. Hool. 1966. 26 pages.
- No. 13. A Method of Estimation of Gross Yield of Douglas-Fir. By Robert O. Curtis. 1967. 24 pages.
- No. 14. The European Pine Shoot Moth—Ecology and Control in the Lake States. By William E. Miller. 1967. 72 pages.
- No. 15. The Analysis of Numerical Change in Gypsy Moth Populations. By Robert W. Campbell. 1967. 33 pages.
- No. 16. Allocating Funds to Timber Management Research. By James E. Bethune and Jerome L. Clutter. 1969. 22 pages.
- No. 17. Dynamics and Simulated Yield of Douglas-Fir. By Kenneth J. Mitchell. 1975. 39 pages.
- No. 18. Techniques for Prescribing Optimal Timber Harvest and Investment Under Different Objectives—Discussion and Synthesis. By K. Norman Johnson and H. Lynn Scheurman. 1977. 31 pages.
- No. 19. Forest Stand Responses to Defoliation by the Gypsy Moth. By Robert W. Campbell and Ronald J. Sloan. 1977. 34 pages.
- No. 20. Temporal and Spatial Variations in the Water Status of Forest Trees. By T. M. Hinckley, J. P. Lassoie, and S. W. Running. 1978. 72 pages.
- No. 21. Early Revegetation and Nutrient Dynamics Following the 1971 Little Sioux Forest Fire in Northeastern Minnesota. By Lewis F. Ohmann and David F. Grigal. 1979. 80 pages.
- No. 22. The 1980 Softwood Timber Assessment Market Model: Structure, Projections, and Policy Simulations. By Darius M. Adams and Richard W. Haynes. 1980. 76 pages. (Out of print.)
- No. 23. Genetic Variation in Seedling Progeny of Ponderosa Pine Provenances. By Ralph A. Read. 1980. 59 pages.

*(Cont'd on outside back cover)*

No. 24. Root and Root System Terminology. By R. F. Sutton and R. W. Tinus. 1983. 137 pages.

No. 25. Commercial Vegetative Inoculum of *Pisolithus tinctorius* and Inoculation Techniques for Development of Ectomycorrhizae on Bare-root Tree Seedlings. By D. H. Marx, C. E. Cordell, D. S. Kenney, J. G. Mexal, J. D. Artman, J. W. Riffle, and R. J. Molina. 1984. 101 pages.

No. 26. Predicting Regeneration in the Grand Fir-Cedar-Hemlock Ecosystem of the Northern Rocky Mountains. By Dennis E. Ferguson, Albert R. Stage, and Raymond J. Boyd. 1986. 41 pages.